Next Iterations

		x_1	x_2	x_3	x_4	x_5	x_6
	120	0	-4	0	2	4	0
$x_3 =$	10	0	1.5	1	1	-0.5	0
$x_1 =$	0	1	-1	0	-1	1	0
$x_6 =$	10	0	2.5	0	1	-1.5	1

- $\bar{c}_2, \bar{c}_3 < 0 \implies$ two possible choices for pivot column.
- Choose x_3 to enter the new basis.
- $u_3 < 0 \implies$ third row cannot be chosen as pivot row.
- Choose x_4 to leave basis.
- New basic feasible solution $(0, 0, 10, 0, 0, 10)^T$ with cost -120, corresponding to point B in the original polyhedron.

Geometric Interpretation in the Original Polyhedron

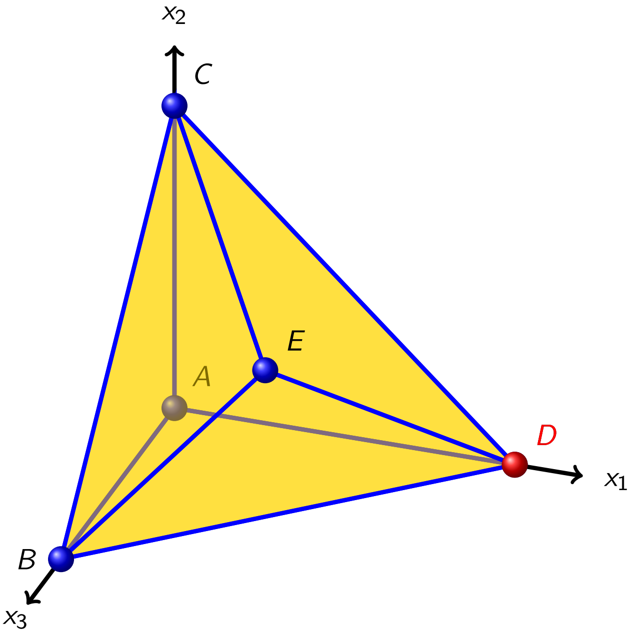
$A = (0, 0, 0)^T$

$B = (0, 0, 10)^T$

$C = (0, 10, 0)^T$

$D = (10, 0, 0)^T$

$E = (4, 4, 4)^T$



Geometric Interpretation in the Original Polyhedron

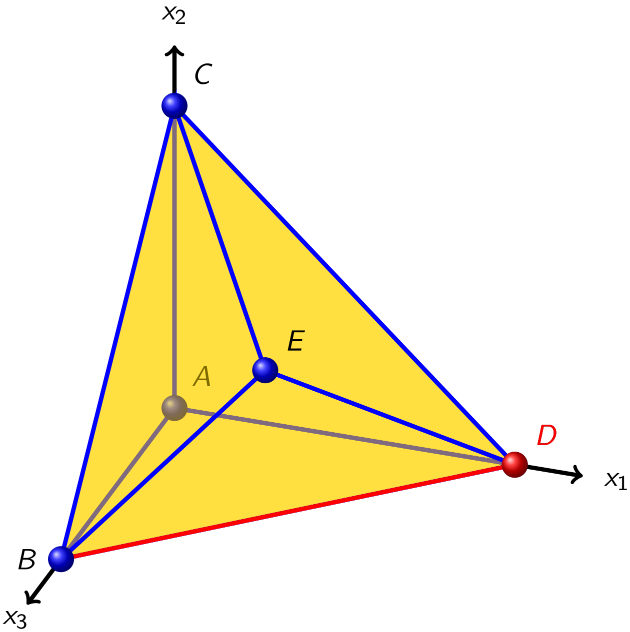
$A = (0, 0, 0)^T$

$B = (0, 0, 10)^T$

$C = (0, 10, 0)^T$

$D = (10, 0, 0)^T$

$E = (4, 4, 4)^T$



Geometric Interpretation in the Original Polyhedron

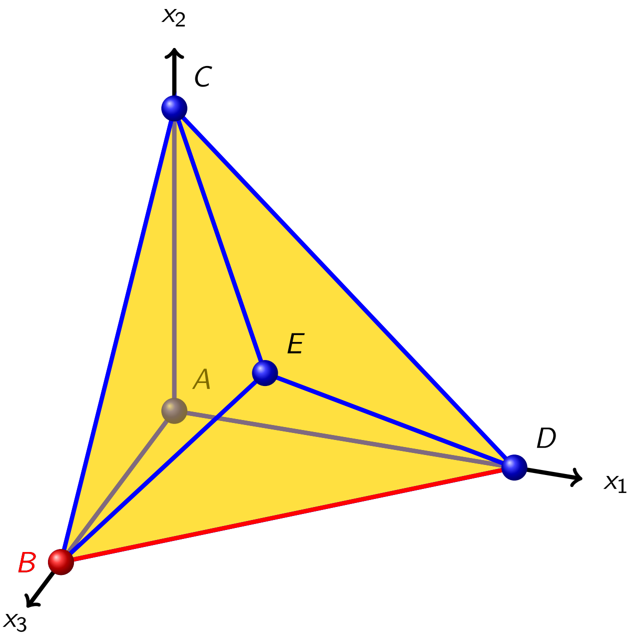
$A = (0, 0, 0)^T$

$B = (0, 0, 10)^T$

$C = (0, 10, 0)^T$

$D = (10, 0, 0)^T$

$E = (4, 4, 4)^T$



Geometric Interpretation in the Original Polyhedron

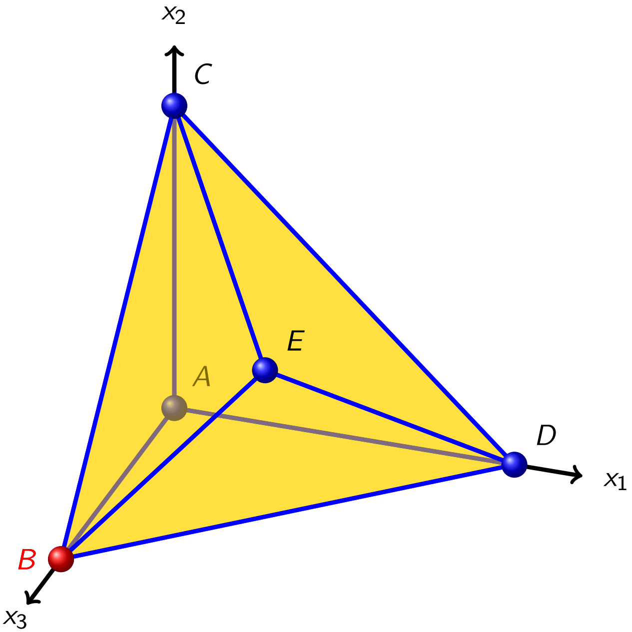
$A = (0, 0, 0)^T$

$B = (0, 0, 10)^T$

$C = (0, 10, 0)^T$

$D = (10, 0, 0)^T$

$E = (4, 4, 4)^T$



Next Iterations

		x_1	x_2	x_3	x_4	x_5	x_6
	120	0	−4	0	2	4	0
$x_3 =$	10	0	1.5	1	1	−0.5	0
$x_1 =$	0	1	−1	0	−1	1	0
$x_6 =$	10	0	2.5	0	1	−1.5	1

$x_2 \leq \frac{10}{1.5} = \frac{20}{3}$

Next Iterations

		x_1	x_2	x_3	x_4	x_5	x_6
	120	0	-4	0	2	4	0
$x_3 =$	10	0	1.5	1	1	-0.5	0
$x_1 =$	0	1	-1	0	-1	1	0
$x_6 =$	10	0	2.5	0	1	-1.5	1

Next Iterations

		x_1	x_2	x_3	x_4	x_5	x_6	$\frac{x_{B(i)}}{u_i}$
	120	0	-4	0	2	4	0	
$x_3 =$	10	0	1.5	1	1	-0.5	0	$\frac{20}{3}$
$x_1 =$	0	1	-1	0	-1	1	0	—
$x_6 =$	10	0	2.5	0	1	-1.5	1	4

Next Iterations

		x_1	x_2	x_3	x_4	x_5	x_6	$\frac{x_{B(i)}}{u_i}$
	120	0	-4	0	2	4	0	
$x_3 =$	10	0	1.5	1	1	-0.5	0	$\frac{20}{3}$
$x_1 =$	0	1	-1	0	-1	1	0	$-$
$x_6 =$	10	0	2.5	0	1	-1.5	1	4 $< \frac{20}{3}$

Next Iterations

		x_1	x_2	x_3	x_4	x_5	x_6
	120	0	-4	0	2	4	0
$x_3 =$	10	0	1.5	1	1	-0.5	0
$x_1 =$	0	1	-1	0	-1	1	0
$x_6 =$	10	0	2.5	0	1	-1.5	1

x_2 enters the basis, x_6 leaves it.

Next Iterations

		x_1	x_2	x_3	x_4	x_5	x_6
	120	0	−4	0	2	4	0
$x_3 =$	10	0	1.5	1	1	−0.5	0
$x_1 =$	0	1	−1	0	−1	1	0
$x_6 =$	10	0	2.5	0	1	−1.5	1

x_2 enters the basis, x_6 leaves it. We get

		x_1	x_2	x_3	x_4	x_5	x_6
	136	0	0	0	3.6	1.6	1.6
$x_3 =$	4	0	0	1	0.4	0.4	−0.6
$x_1 =$	4	1	0	0	−0.6	0.4	0.4
$x_2 =$	4	0	1	0	0.4	−0.6	0.4

Next Iterations

$\min\ -136 + 3.6x_4 + 1.6x_5 + 1.6x_6$ s.t. $x_i \geq 0$

	x_1	x_2	x_3	x_4	x_5	x_6
120	0	-4	0	2	4	0
$x_3 =$ 10	0	1.5	1	1	-0.5	0
$x_1 =$ 0	1	-1	0	-1	1	0
$x_6 =$ 10	0	2.5	0	1	-1.5	1

x_2 enters the basis, x_6 leaves it. We get

	x_1	x_2	x_3	x_4	x_5	x_6
136	0	0	0	3.6	1.6	1.6
$x_3 =$ 4	0	0	1	0.4	0.4	-0.6
$x_1 =$ 4	1	0	0	-0.6	0.4	0.4
$x_2 =$ 4	0	1	0	0.4	-0.6	0.4

and the reduced costs are all non-negative.

Next Iterations

		x_1	x_2	x_3	x_4	x_5	x_6
	120	0	-4	0	2	4	0
$x_3 =$	10	0	1.5	1	1	-0.5	0
$x_1 =$	0	1	-1	0	-1	1	0
$x_6 =$	10	0	2.5	0	1	-1.5	1

x_2 enters the basis, x_6 leaves it. We get

		x_1	x_2	x_3	x_4	x_5	x_6
	136	0	0	0	3.6	1.6	1.6
$x_3 =$	4	0	0	1	0.4	0.4	-0.6
$x_1 =$	4	1	0	0	-0.6	0.4	0.4
$x_2 =$	4	0	1	0	0.4	-0.6	0.4

and the reduced costs are all non-negative.

Thus $(4, 4, 4, 0, 0, 0)$ is an optimal solution with cost -136, corresponding to point $E = (4, 4, 4)$ in the original polyhedron.

Next Iterations

		x_1	x_2	x_3	x_4	x_5	x_6
	120	0	-4	0	2	4	0
$x_3 =$	10	0	1.5	1	1	-0.5	0
$x_1 =$	0	1	-1	0	-1	1	0
$x_6 =$	10	0	2.5	0	1	-1.5	1

x_2 enters the basis, x_6 leaves it. We get

		x_1	x_2	x_3	x_4	x_5	x_6
	136	0	0	0	3.6	1.6	1.6
$x_3 =$	4	0	0	1	0.4	0.4	-0.6
$x_1 =$	4	1	0	0	-0.6	0.4	0.4
$x_2 =$	4	0	1	0	0.4	-0.6	0.4

and the reduced costs are all non-negative.

Thus $(4, 4, 4, 0, 0, 0)$ is an optimal solution with cost -136, corresponding to point $E = (4, 4, 4)$ in the original polyhedron. Why is this optimal?

All Iterations from Geometric Point of View

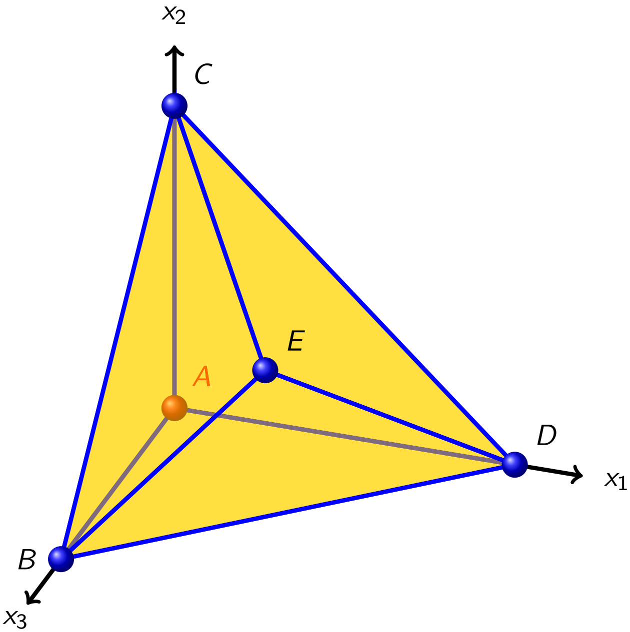
$A = (0, 0, 0)^T$

$B = (0, 0, 10)^T$

$C = (0, 10, 0)^T$

$D = (10, 0, 0)^T$

$E = (4, 4, 4)^T$



All Iterations from Geometric Point of View

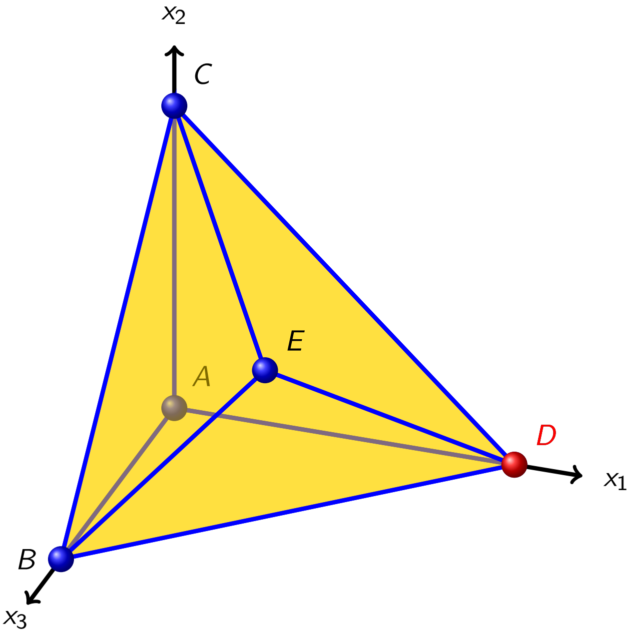
$A = (0, 0, 0)^T$

$B = (0, 0, 10)^T$

$C = (0, 10, 0)^T$

$D = (10, 0, 0)^T$

$E = (4, 4, 4)^T$



All Iterations from Geometric Point of View

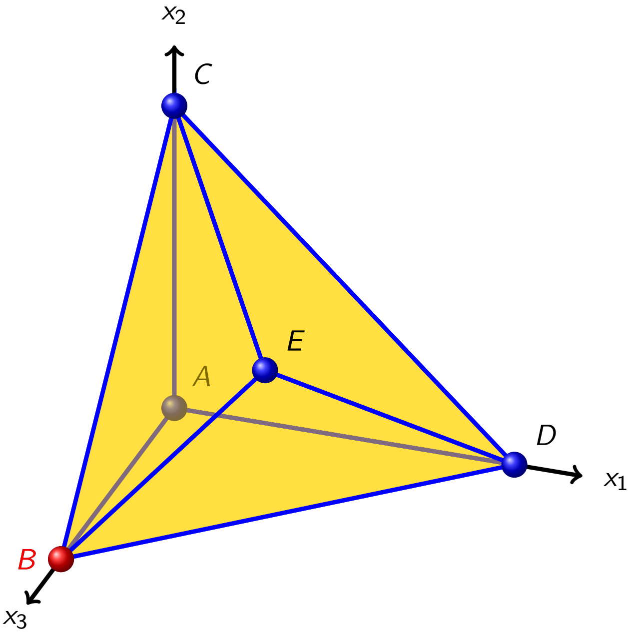
$A = (0, 0, 0)^T$

$B = (0, 0, 10)^T$

$C = (0, 10, 0)^T$

$D = (10, 0, 0)^T$

$E = (4, 4, 4)^T$



All Iterations from Geometric Point of View

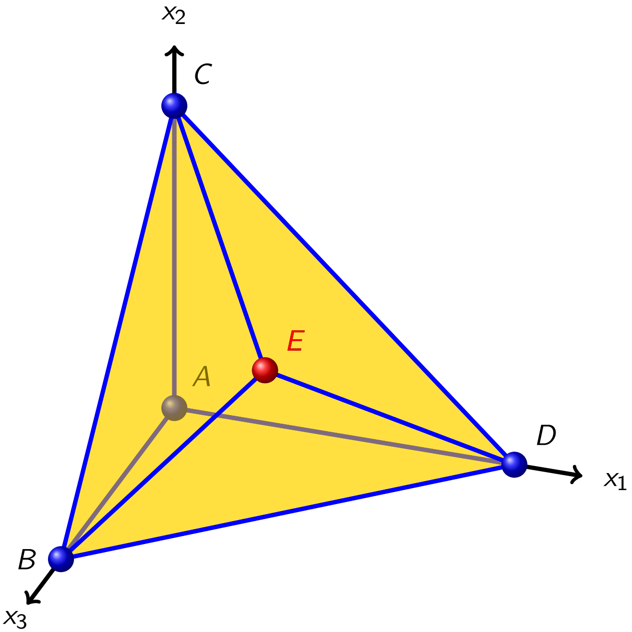
$A = (0, 0, 0)^T$

$B = (0, 0, 10)^T$

$C = (0, 10, 0)^T$

$D = (10, 0, 0)^T$

$E = (4, 4, 4)^T$



Cycling

Problem: If an LP is degenerate, the simplex method might end up in an infinite loop (cycling).

Cycling

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Example:

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	3	$-3/4$	20	$-1/2$	6	0	0	0
$x_5 =$	0	$1/4$	-8	-1	9	1	0	0
$x_6 =$	0	$1/2$	-12	$-1/2$	3	0	1	0
$x_7 =$	1	0	0	1	0	0	0	1

Cycling

Problem: If an LP is degenerate, the simplex method might end up in an infinite loop (cycling).

Example:

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	3	$-3/4$	20	$-1/2$	6	0	0	0
$x_5 =$	0	$1/4$	-8	-1	9	1	0	0
$x_6 =$	0	$1/2$	-12	$-1/2$	3	0	1	0
$x_7 =$	1	0	0	1	0	0	0	1

Pivoting rules

- Column selection: let nonbasic variable with most negative reduced cost $\bar{c}_j$ enter the basis, i. e., steepest descent rule.
- Row selection: among basic variables that are eligible to exit the basis, select the one with smallest subscript.

Iteration 1

:

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	3	$-3/4$	20	$-1/2$	6	0	0	0
$x_5 =$	0	$1/4$	-8	-1	9	1	0	0
$x_6 =$	0	$1/2$	-12	$-1/2$	3	0	1	0
$x_7 =$	1	0	0	1	0	0	0	1

Bases visited

$(5, 6, 7)$

Iteration 1

:

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	3	$-3/4$	20	$-1/2$	6	0	0	0
$x_5 =$	0	$1/4$	-8	-1	9	1	0	0
$x_6 =$	0	$1/2$	-12	$-1/2$	3	0	1	0
$x_7 =$	1	0	0	1	0	0	0	1

Bases visited

$(5, 6, 7)$

Iteration 1

:

		x_1	x_2	x_3	x_4	x_5	x_6	x_7	$\frac{x_{B(i)}}{u_i}$
	3	$-3/4$	20	$-1/2$	6	0	0	0	
$x_5 =$	0	$1/4$	-8	-1	9	1	0	0	0
$x_6 =$	0	$1/2$	-12	$-1/2$	3	0	1	0	0
$x_7 =$	1	0	0	1	0	0	0	1	$-$

Bases visited

$(5, 6, 7)$

Iteration 1

:

		x_1	x_2	x_3	x_4	x_5	x_6	x_7	$\frac{x_{B(i)}}{u_i}$
	3 3	$-3/4$ 0	20 -4	$-1/2$ -3.5	6 33	0 3	0 0	0 0	
$x_5 =$	0	$1/4$	-8	-1	9	1	0	0	0
$x_6 =$	0	$1/2$	-12	$-1/2$	3	0	1	0	0
$x_7 =$	1	0	0	1	0	0	0	1	–

Basis change: x_1 enters the basis x_5 leaves.

Bases visited

$(5, 6, 7)$

Iteration 1

:

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	3	$-3/4$	20	$-1/2$	6	0	0	0
$x_5 =$	0	$1/4$	-8	-1	9	1	0	0
$x_6 =$	0	$1/2$	-12	$-1/2$	3	0	1	0
$x_7 =$	1	0	0	1	0	0	0	1

Basis change: x_1 enters the basis x_5 leaves.

Bases visited

$(5, 6, 7)$

Iteration 1

:

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	3	0	-4	$-7/2$	33	3	0	0
$x_5 =$	0	$1/4$	-8	-1	9	1	0	0
$x_6 =$	0	$1/2$	-12	$-1/2$	3	0	1	0
$x_7 =$	1	0	0	1	0	0	0	1

Basis change: x_1 enters the basis x_5 leaves.

Bases visited

$(5, 6, 7)$

Iteration 1

:

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	3	0	-4	$-7/2$	33	3	0	0
$x_5 =$	0	$1/4$	-8	-1	9	1	0	0
$x_6 =$	0	$1/2$	-12	$-1/2$	3	0	1	0
$x_7 =$	1	0	0	1	0	0	0	1

Basis change: x_1 enters the basis x_5 leaves.

Bases visited

$(5, 6, 7)$

Iteration 1

:

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	3	0	-4	$-7/2$	33	3	0	0
$x_5 =$	0	$1/4$	-8	-1	9	1	0	0
$x_6 =$	0	0	4	$3/2$	-15	-2	1	0
$x_7 =$	1	0	0	1	0	0	0	1

Basis change: x_1 enters the basis x_5 leaves.

Bases visited

$(5, 6, 7)$

Iteration 1

:

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	3	0	-4	$-7/2$	33	3	0	0
$x_5 =$	0	$1/4$	-8	-1	9	1	0	0
$x_6 =$	0	0	4	$3/2$	-15	-2	1	0
$x_7 =$	1	0	0	1	0	0	0	1

Basis change: x_1 enters the basis x_5 leaves.

Bases visited

$(5, 6, 7)$

Iteration 1

:

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	3	0	-4	$-7/2$	33	3	0	0
$x_5 =$	0	$1/4$	-8	-1	9	1	0	0
$x_6 =$	0	0	4	$3/2$	-15	-2	1	0
$x_7 =$	1	0	0	1	0	0	0	1

Basis change: x_1 enters the basis x_5 leaves.

Bases visited

$(5, 6, 7)$

Iteration 1

:

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	3	0	-4	$-7/2$	33	3	0	0
$x_1 =$	0	1	-32	-4	36	4	0	0
$x_6 =$	0	0	4	$3/2$	-15	-2	1	0
$x_7 =$	1	0	0	1	0	0	0	1

Basis change: x_1 enters the basis x_5 leaves.

Bases visited

$(5, 6, 7)$

Iteration 2

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	3	0	-4	$-7/2$	33	3	0	0
$x_1 =$	0	1	-32	-4	36	4	0	0
$x_6 =$	0	0	4	$3/2$	-15	-2	1	0
$x_7 =$	1	0	0	1	0	0	0	1

Bases visited

$(5, 6, 7) \rightarrow (1, 6, 7)$