

Tutorials
Optimisation
2018
Exercise Sheet 7

Exercise 12:

Consider the following linear program:

$$\begin{array}{rcllcl} \min & x_1 & + & 2x_2 & + & 5x_3 & + & 2x_4 & & \\ s.t. & 2x_1 & + & x_2 & + & x_3 & + & x_4 & = & 10 \\ & 2x_1 & + & 5x_2 & + & x_3 & & & = & 6 \\ & x_1 & + & x_2 & & & & & = & 1 \\ & & & x_2 & + & x_3 & + & x_4 & \geq & 5 \\ & & & x_1, & x_2, & x_3, & x_4 & \geq & 0 \end{array}$$

- (a) Construct the dual (D) of this LP.
- (b) Verify that $x^* = (0, 1, 1, 8)$ is optimal, using complementary slackness.

Exercise 13:

Consider the example discussed in the lecture (slides 61 – 66).

<http://cgi.csc.liv.ac.uk/~gairing/COMP557/board/20181011.pdf>

<http://cgi.csc.liv.ac.uk/~gairing/COMP557/board/20181016.pdf>

- (a) Compute B^{-1} of basis 2 (module slide 62) and give the associated dual basic solution.
- (b) Do the same for basis 3.

Exercise 14:

Consider the following linear program:

$$\begin{array}{rccccrcrcl} \min & 9x_1 & + & x_2 & + & 3x_3 & & & & \\ s.t. & 2x_1 & + & 3x_2 & - & x_3 & + & x_4 & = & 3 \\ & 4x_1 & + & & + & 4x_3 & - & 2x_4 & = & 4 \\ & -x_1 & - & 2x_2 & + & 2x_3 & + & x_4 & = & -1 \\ & & & & & x_1, & x_2, & x_3, & x_4 & \geq & 0 \end{array}$$

- (a) Construct the dual (D) of this LP.
- (b) Verify that $x^* = (\frac{1}{3}, 1, \frac{2}{3}, 0)$ is optimal using complementary slackness.
- (c) Let B be the basis consisting of columns 1,2, and 4.
- Compute B^{-1} .
 - Use B^{-1} to compute the associated primal and dual basic solutions (cf. Observation 5.7, slide 155) and state (and provide an argument) whether they are feasible or not.