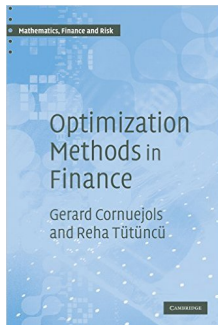


COMP331/557

Chapter 5: Optimisation in Finance: Cash-Flow

(Cornuejols & Tütüncü, Chapter 3)



Cash-Flow Management Problem

A company has the following net cash flow requirements (in 1000's of £):

Month	Jan	Feb	Mar	Apr	May	Jun
Net cash flow	-150	-100	200	-200	50	300

E.g.: In January we have to pay £150k and in March we get £200k.

Initially we have no cash but the following possibilities to borrow/invest money:

- i** a line of credit of up to £100k at an interest rate of 1% per month;
- ii** in any one of the first three months, it can issue 90-day commercial paper bearing a total interest of 2% for the three-month period;
- iii** excess funds can be invested at an interest rate of 0.3% per month.

Task: We want to maximise the companies wealth in June, while fulfilling all payments.

Cash-Flow Management Problem – Modelling as LP

Decision Variables

- ▶ v .. wealth in June
- ▶ x_i .. amount drawn from credit line in month i
- ▶ y_i .. amount of commercial paper issued in month i
- ▶ z_i .. excess funds in month i

LP formulation:

$$\begin{array}{ll} \max & v \\ \text{s.t.} & x_1 + y_1 - z_1 = 150 \\ & x_2 + y_2 - 1.01x_1 + 1.003z_1 - z_2 = 100 \\ & x_3 + y_3 - 1.01x_2 + 1.003z_2 - z_3 = -200 \\ & x_4 - 1.02y_1 - 1.01x_3 + 1.003z_3 - z_4 = 200 \\ & x_5 - 1.02y_2 - 1.01x_4 + 1.003z_4 - z_5 = -50 \\ & -1.02y_3 - 1.01x_5 + 1.003z_5 - v = -300 \\ & x_i \leq 100 \quad \forall i \\ & x_i, y_i, z_i \geq 0 \quad \forall i \end{array}$$

Cash-Flow Management Problem – Modelling as LP

cashflow.lp

Maximize

wealth: v

Subject To

Jan: $x_1 + y_1 - z_1 = 150$

Feb: $x_2 + y_2 - 1.01 x_1 + 1.003 z_1 - z_2 = 100$

Mar: $x_3 + y_3 - 1.01 x_2 + 1.003 z_2 - z_3 = -200$

Apr: $x_4 - 1.02 y_1 - 1.01 x_3 + 1.003 z_3 - z_4 = 200$

May: $x_5 - 1.02 y_2 - 1.01 x_4 + 1.003 z_4 - z_5 = -50$

Jun: $-1.02 y_3 - 1.01 x_5 + 1.003 z_5 - v = -300$

Bounds

$0 \leq x_1 \leq 100$

$0 \leq x_2 \leq 100$

$0 \leq x_3 \leq 100$

$0 \leq x_4 \leq 100$

$0 \leq x_5 \leq 100$

$-\text{Inf} \leq v \leq \text{Inf}$

End

Cash-Flow Management Problem – Modelling as LP

Gurobi Output

```
Solved in 5 iterations and 0.00 seconds
Optimal objective 9.249694915e+01
v 92.4969491525
x1 0.0
y1 150.0
z1 0.0
x2 0.0
y2 100.0
z2 0.0
x3 0.0
y3 151.944167498
z3 351.944167498
x4 0.0
z4 0.0
x5 52.0
z5 0.0

Obj: 92.4969491525
```

Optimal Investment Strategy:

Jan: Issue commercial paper for £150k.

Feb: Issue commercial paper for £100k.

Mar: Issue paper for \approx £152k and invest \approx £352k.

Apr: Take excess to pay outgoing cashflow.

May: Take a credit of £52k

Jun: wealth \approx £92k