Good-For-Games Automata

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Based on work of
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and joint work with
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Highlights Spotlight Workshop *2020*

Quick State of Affairs

- Good for games (GFG) automata lie in between deterministic and nondeterministic/alternating automata.
- They were defined in 2006 and have recently got a lot of attention.
- They provide a potential for breakthroughs in formal methods, especially in synthesis and game solving.
- This potential was not yet realized…

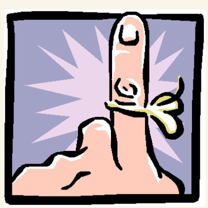
Agenda

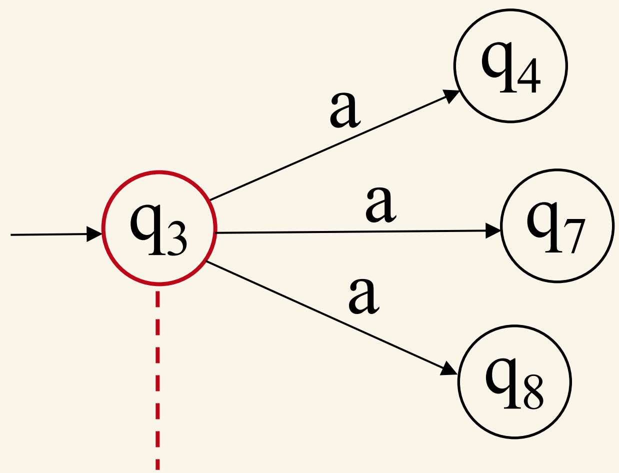
- GFG Automata – Definition(s)
- In the Service of Synthesis
- In the Service of Game Solving
- What We Know and What We Don’t
- The Road Ahead

GFG Automata – Definition(s)

Nondeterminism

- A nondeterministic automaton “decides” which transition to choose at every position.
- The automaton may “see” the whole past and the whole future.

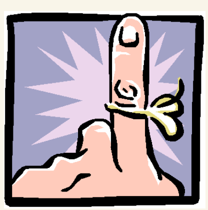


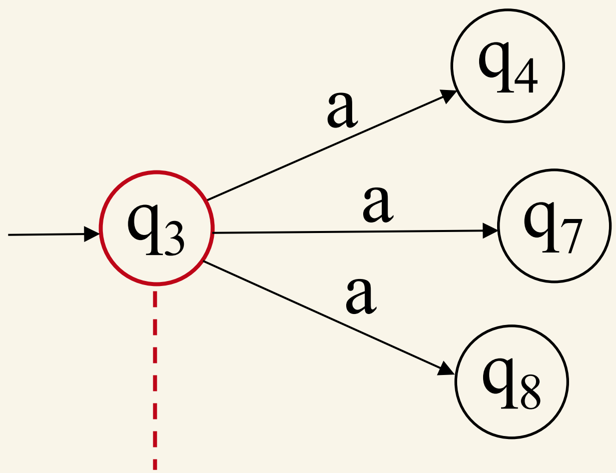


w = “aaababbbbbaaa a abbbabababbababa………….…”

History Determinism

- A nondeterministic automaton *A* is *history-deterministic* if it can "decide" by the past alone: It can accept all words of its language *L(A)*, following a strategy $\boldsymbol{\sigma}$:word-read $\rightarrow$ next state.





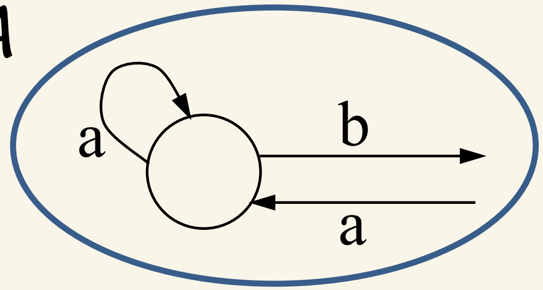
w = "aaababbbbbaaa a

Good For Gameness

- An automaton *A* is *good for games* (GFG) if it composes well with games: For every game G with winning condition L(A), the game G×A has the same winner as G.

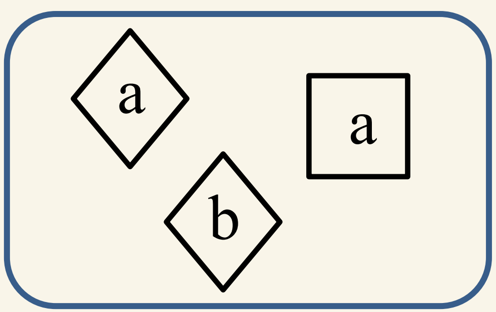
Game *G* between □-Adam and ◊-Eve. Σ-Labeled positions/trans. Eve wins a play if the generated word is in L(A).

A



Automaton *A* over alphabet Σ with acceptance condition α.

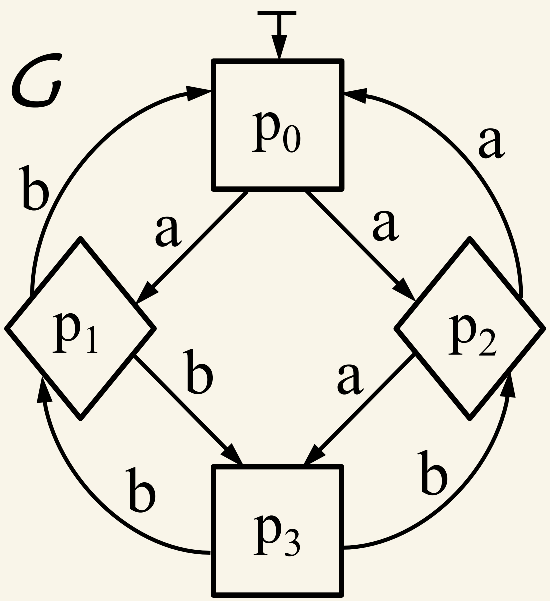
G



G×A

Game between □-Adam and ◊-Eve. Winning condition α.

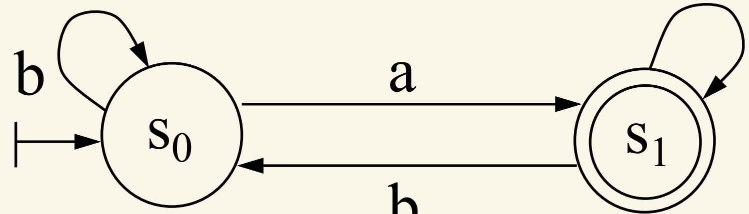
Game × Automaton



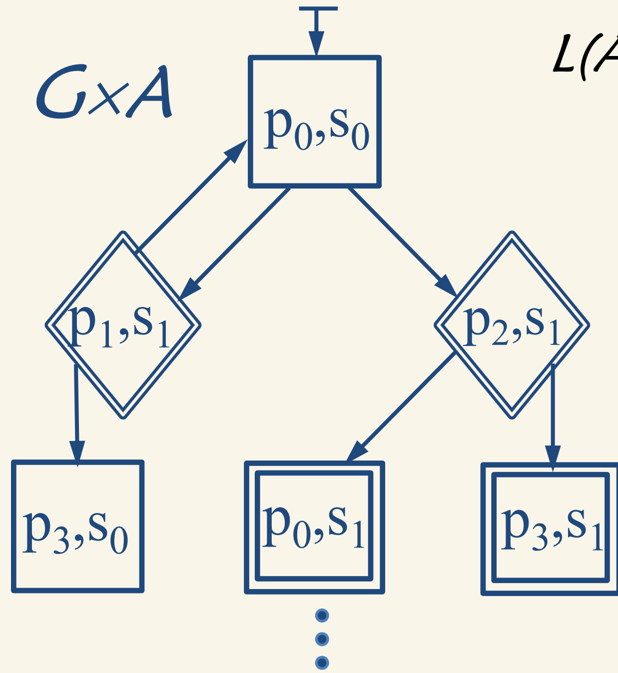
□-Adam and ◊-Eve.
G's (Eve) Winning condition is L(A).

Every deterministic automaton is GFG

Büchi automaton A



L(A) = Infinitely many a's

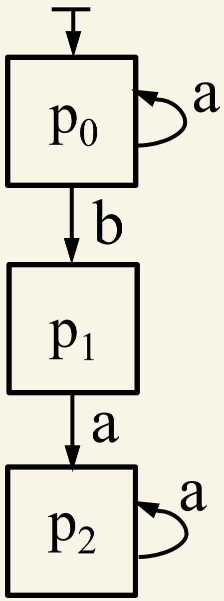


A is GFG: Adam and Eve play the same in G and in G×A. (Looking at the p_i part of positions.)

(G×A)'s winning condition is Büchi

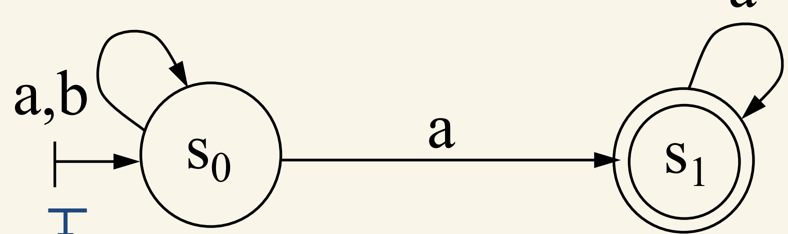
Game × Automaton

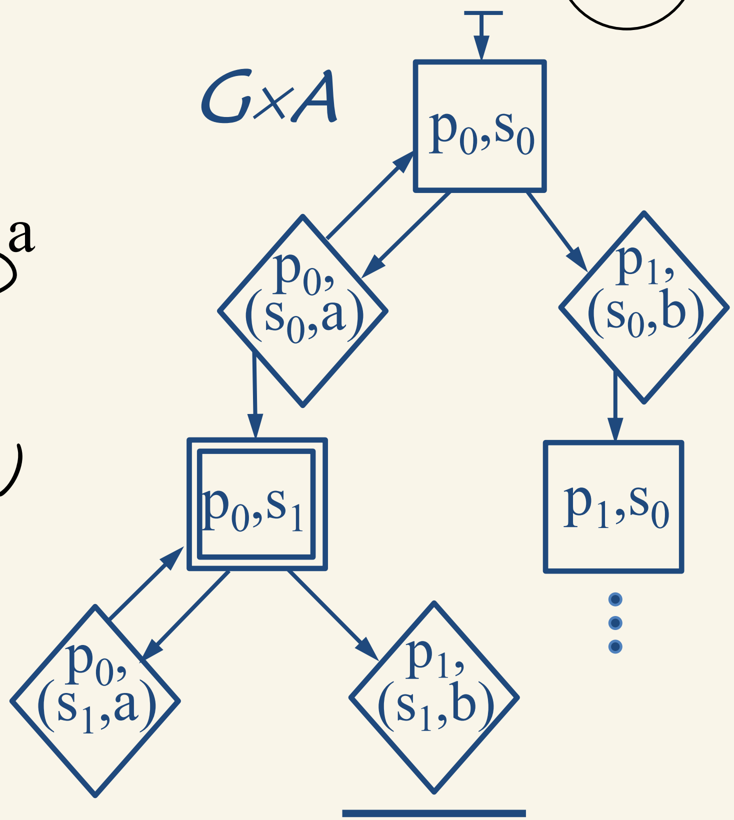
G



G's Winning condition is L(A)

Büchi automaton *A*





L(A) = Finitely many b's

Eve wins *G*, but loses *G×A* !

Not every nondeterministic automaton is GFG

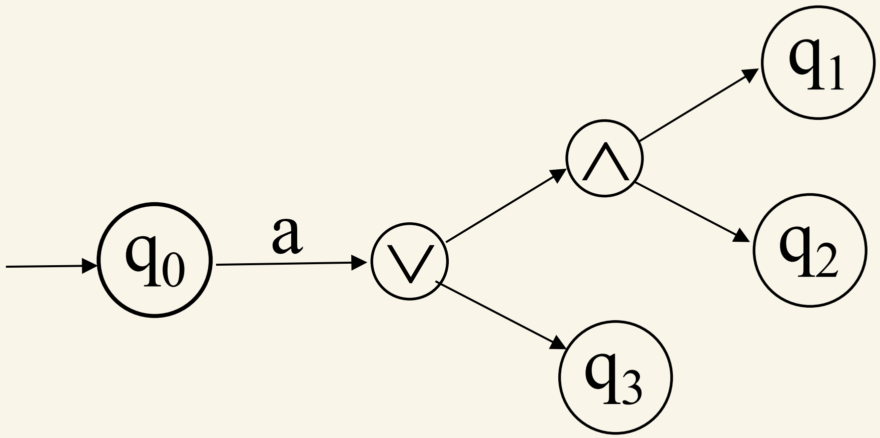
Extending

History Determinism and Good for Gamesness

from nondeterministic to alternating automata

Alternation

- In an alternating automaton there is both nondeterminism and universality.
- A run can be viewed as a game between Eve, who resolves the nondeterminism and wants to accept the word, and Adam who resolves the universality and wants to reject the word.



History Determinism

- An alternating automaton *A* is *history-deterministic* if both Eve and Adam can follow strategies and accept (respectively reject) all words in (respectively out of) the language *L(A)*.

- If only one of them has such a strategy, the automaton is *half-history-deterministic* (with respect to nondeterminism / universality).

Good For Gameness

- The definition is exactly the same as for nondeterminism:
  An automaton *A* is *good for games* (GFG) if it composes well with games: For every game G with winning condition L(A), the game G×A has the same winner as G.
- The definition of the synchronized product G×A is extended – In the product game, Eve controls the nondeterminism within the transitions and Adam controls the universality within them.

Equivalent Definitions

Robust Notion

A nondeterministic/alternating automaton is

History Deterministic [Colcombet, 2009]

iff

Good for Games [Henzinger & Piterman 2006]

iff

Adaquate to Letter Games [Henzinger & Piterman 2006]

iff

Good for Composition with Alternating Automata
[Colcombet, 2013]

iff

Good for Trees [Kupferman & Safra & Vardi 1996]

Equivalence proofs and extensions to alternating automata –
Partly in the above and the rest in [Boker and Lehtinen 2019]

Agenda

✓ GFG Automata – Definition(s)

In the Service of Synthesis

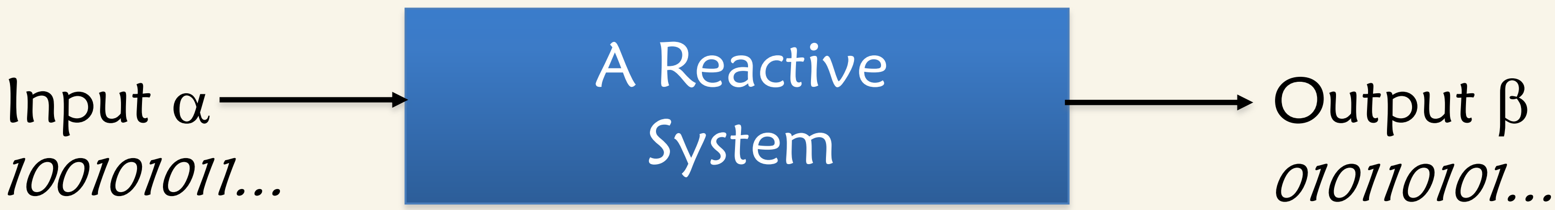
- In the Service of Game Solving
- What We Know and What We Don't
- The Road Ahead

Church's Synthesis Problem

Synthesis: given a specification, decide if there exists a system that *realizes* it, and if there is, automatically construct one.

Reactive synthesis: Synthesis of reactive systems.

- Proposed by Church in 1957; Also called Church's problem.



- The system transforms, letter by letter, an infinite input sequence $\alpha \in (\mathrm{Input})^{\omega}$ into an output sequence $\beta \in (\mathrm{Input})^{\omega}$.
- The pair $(\alpha,\beta) \in (I \times O)^{\omega}$ (or $\in (I \cup O)^{\omega}$) should satisfy the specification, which is usually a logic formula φ.

Synthesis – Complexity

- Büchi and Landweber showed in 1969 that it is decidable for ω-regular specifications.
- Pnueli and Rosner showed in 1989 that for LTL specifications it is 2EXPTIME complete.
- Efficient algorithms for some LTL fragments:
  - Boolean combinations of safety/reachability formulas.
  - Recurrence/persistence formulas.
  - Most notably, GR(1) and some generalizations of it.

- Synthesis has not yet reached a satisfying applicative level.

Solving Synthesis

There are two, fundamentally equivalent, approaches for solving reactive synthesis.

- The first is to reduce the synthesis problem to the problem of solving two-player turn-based games.
  - In this process the specification is first translated to an equivalent deterministic *or GFG* automaton.
- The second is to reduce it to the emptiness problem of tree automata.

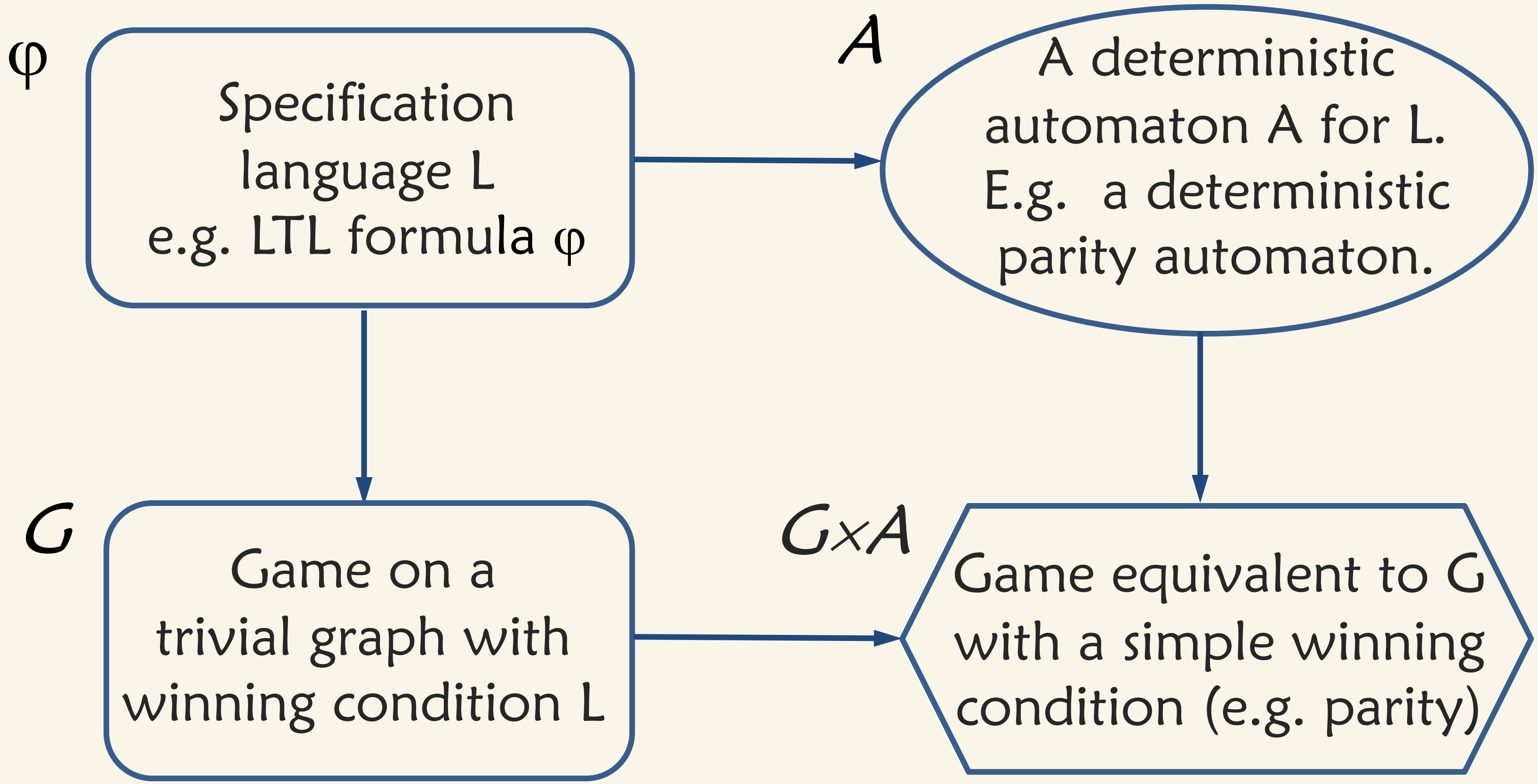
We follow the first approach, in which GFG automata may play a central role,

Synthesis as a Game

Church's synthesis problem is exactly a two-player turn-base win-lose infinite game:

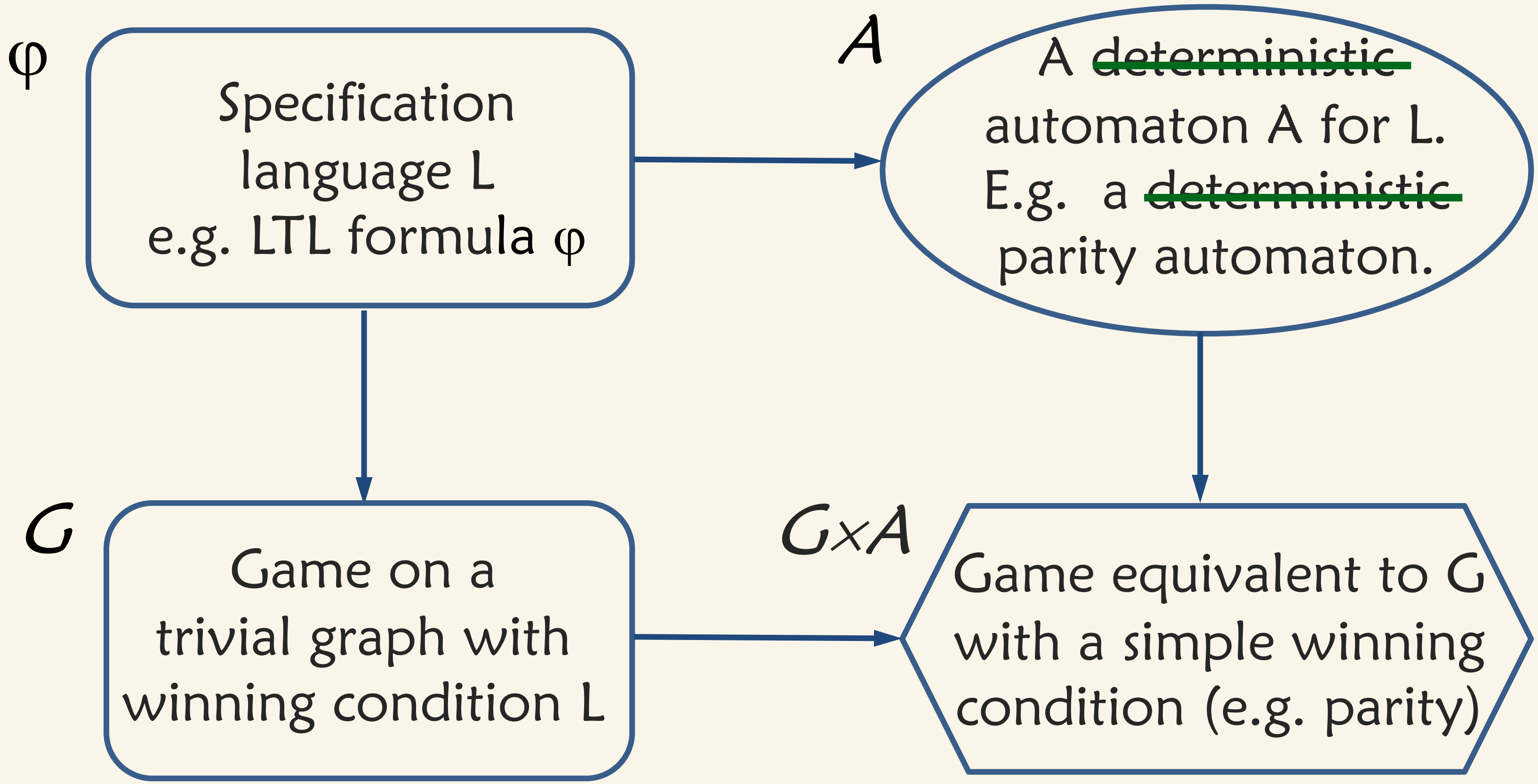
- The environment, producing the input, is Player I – Adam.
- The system, producing the output, is player II – Eve.
- The input and output letters are the players' actions.
- The specification language is the winning condition.
- There exists a transducer iff Eve wins the game.
- The required system is a description of Eve's winning strategy.

Solving the Synthesis Game



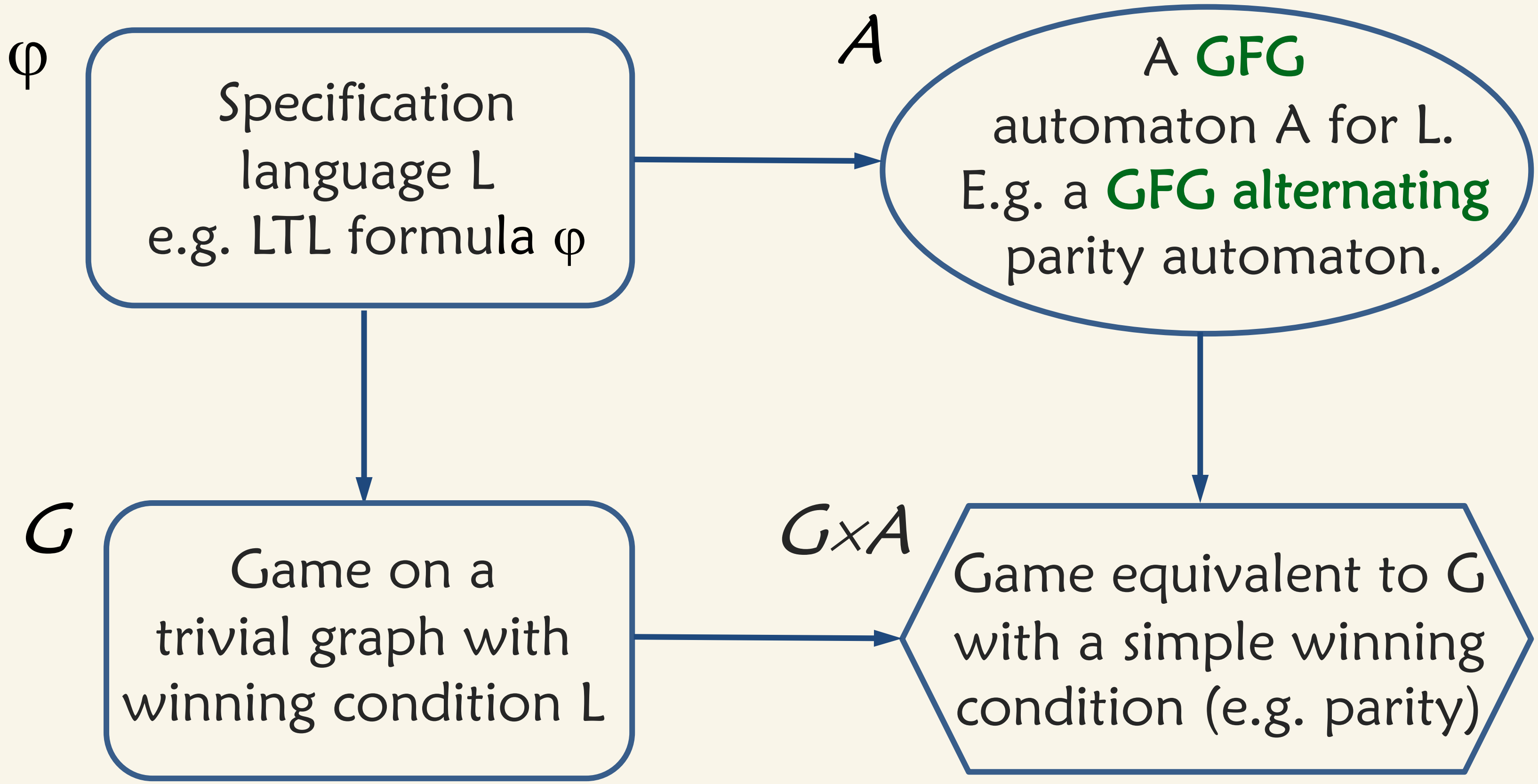
Problem: Deterministic automata are big (double-exp from LTL)

Solving the Synthesis Game



~~Problem: Deterministic automata are big (double-exp from LTL)~~

Solving the Synthesis Game



Challenge: Can GFG automata improve the process?

GFG Automata – Challenges

Can GFG automata be

- Substantially different from deterministic automata?
- More expressive than them?
- More concise?
- Better symbolically represented?
- Concicely translated from useful logics?
  - From interesting fragments of LTL? of μ-calculus?

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- ✓ GFG Automata – Definition(s)
- ✓ In the Service of Synthesis

In the Service of Game Solving

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Quasipolynomial Era

- Since 2017 when Calude, Jain, Khoussainov, Li, and Stephan brought down the upper bound for solving parity games to quasipolynomial, the automata-theoretical aspects of solving parity games have been studied in more depth.
- In particular, Bojańczyk and Czerwiński and Czerwiński et al describe the quasipolynomial algorithms for solving parity games explicitly in terms of deterministic word automata that *separate* some word languages.

Better than Quasipolynomial?

- A polynomial deterministic *or GFG* such automata would imply a polynomial algorithm for parity games (and for other two-player turn-based games on graphs).
- However, Czerwiński at al. showed that the smallest possible such *nondeterministic* automaton is quasipolynomial.
- Can alternating GFG automata allow to use the separation approach for achieving better algorithms?

Good for Small Game

- Lehtinen's recent quasipolynomial algorithm for solving parity games uses an intermediate *nondeterministic* automaton that might not be deterministic nor GFG

- Yet, it is *good for small games*, in the sense that it can be properly composed with games of some bounded size.

- Can alternating good for small games automata allow to further improve improve algorithms for solving two-player turn-based games on graphs?

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Expressiveness

GFG automata vs. deterministic automata

- Regular automata: same expressiveness [KSV96, NW98]
- ω-regular autotomata: same expressiveness [KSV96, NW98]
- Cost automata: GFG are as expressive as nondererminisitc, strictly more than deterministic [Col09]
- ω-PDAs: GFG are more than deterministic and less then nondeterministic. Decidable universality. [LZ20]
- Timed-, limit-average- discounted-sum- register- automata: Preliminary work by Henzinger, Lehtinen and Totzke

Succinctness

- GFG automata over finite words always embody deterministic ones [KSV96]
- When GFG automata were defined [HP06], all examples of GFG automata embodied deterministic ones.
  - The motivation was to have a simpler symbolic representation for these bigger automata.
- Colcombet conjectured in 2012 that

It need not (already Büchi and coBüchi) [BKKM13]

Succinctness – Nondeterministic ω-reg

Are nondeterministic GFG ω-regular automata more succinct than deterministic automata of the same type?

- Weak automata: No.
  They embody deterministic automata [KSV96,Mor03]
- Büchi automata: Still open.
  They are up to quadratically more succinct [KS15]
- Co-Büchi automata: Yes, exponentially!
  [KS15]
- Parity and Streett: Yes, exponentially.
  Deterministic parity and Streett automata are co-Büchi type.
- Rabin and Muller: Still open.
  Deterministic Rabin and Muller are not co-Büchi type.

Succinctness – Alternating ω-reg

Are alternating GFG ω-regular automata more succinct than deterministic automata of the same type?

- Weak automata: No. [BL19]
- Büchi and Co-Büchi : Singly exponential. [BL19]
- Parity : Singly exponential. [BKLS20]
- Street, Rabin and Muller: Still open.

GFGness – Nondeterministic ω-reg

Given a nondeterministic ω-regular automaton, what is the complexity of deciding whether it is GFG?

[KS15]: At least as hard as solving games with the same condition

- Weak automata: Polynomial.
  [Lod11] (Unpublished)
- Büchi automata: Polynomial.
  [BK18] Showing equivalence to solving G2 games
- Co-Büchi automata: Polynomial.
  [KS15]; In [BKLS20a] via G2 games
- Parity: ≤ EXPTIME.
  [HP06]; [BK18]: For fixed priorities – the G2 conjecture.
- Rabin, Street and Muller: ≤ EXPTIME.
  [HP06]: Simulation check against a deterministic automaton

GFGness – Alternating ω-reg

Given an alternating ω-regular automaton, what is the complexity of deciding whether it is GFG?

- Weak automata: Polynomial.
  [BKLS20a] Lifting the G2 game to alternating automata
- Büchi and Co-Büchi automata: $\leq$ EXPTIME.
  [BKLS20a] G2 conjecture $\Rightarrow$ polynomial
- Parity: $\leq$ EXPTIME.
  [BKLS20a] For a fixed index, G2 conjecture $\Rightarrow$ polynomial
  Nondeterministic G2 conjecture $\Leftrightarrow$ Alternating G2 conjecture
- Rabin, Streett and Muller: Open.

Logic Formulas → GFG Automata

What interesting/useful logics or fragments of known logics (sucn as LTL and μ-calculus) are succinctly translated to GFG automata?

- Well… so… you know… look … you see… I mean…

  We still know very little

- Iosti & Kuperberg [IK19]: *SvTL* - Eventually safe μ-calculus.
  - Polynomial translation to GFG nondeterministic Co-Büchi.
- Some preliminary work on fragments of LTL by Kupferman, Sickert, and Abu Radi.

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The Road Ahead

The Road Ahead

Thanks

Bring out the potential in GFG automata!

- Expressiveness
  - Can they help with quantitative automata? Other settings?
- Succinctness
  - Alternating ω-regular; stronger conditions; other settings
- Synthesis
  - Interesting logics that succinctly translate to GFG auto.
- Model Checking
  - Interesting logics that succinctly translate to half-GFG auto.
- Game Solving
  - Can GFG or good-for-small game automata help futhur?
- GFGness
  - Complexity? Don't miss Denis' open-problem talk