Brief Announcement: Decidable Graph Languages by Mediated Population Protocols*

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Abstract. We work on an extension of the Population Protocol model of Angluin et al. [1] that allows edges of the communication graph, G, to have *states* that belong to a *constant size set*. In this extension, the so called Mediated Population Protocol model (MPP) [2, 3], both *uniformity* and *anonymity* are preserved. We here study a simplified version of MPP, the Graph Decision Mediated Population Protocol model (GDM), in order to capture MPP's ability to *decide graph languages*. We also prove some first impossibility results both for weakly connected and possibly disconnected communication graphs.

1 The GDM model

A *graph decision mediated population protocol* (GDM) $\mathcal{A}$ consists of a *binary output alphabet* $Y = \{0, 1\}$, a finite set of *agent states* Q, an *agent output function* $O : Q \to Y$ mapping agent states to outputs, a finite set of *edge states* S, an *output instruction* r, a *transition function* $\delta : Q \times Q \times S \to Q \times Q \times S$, an *initial agent state* q_0, and an *initial edge state* s_0.

Let $\mathcal{U}$ denote a *graph universe*, that is, any set of communication graphs. A *graph language* L is a subset of $\mathcal{U}$ containing communication graphs sharing some common property. For example, a common graph universe is the set of all possible directed and weakly connected communication graphs, denoted by $\mathcal{G}$, and $L = \{G \in \mathcal{G} \mid |E(G)| \text{ is even}\}$ is a possible graph language w.r.t. $\mathcal{G}$.

A GDM protocol may run on any graph from a specified graph universe. The graph on which the protocol runs is considered as the *input graph* of the protocol. Note that GDM protocols have no sensed input. Instead, we require each agent in the population to be initially in the initial agent state q_0 and each edge of the communication graph to be initially in the initial edge state s_0. So, the initial network configuration, C_0, of any GDM is defined as $C_0(u) = q_0$, for all $u \in V$, and $C_0(e) = s_0$, for all $e \in E$, and any input graph $G = (V, E)$.

We say that a GDM $\mathcal{A}$ *accepts* an input graph G if in any computation of $\mathcal{A}$ on G after finitely many interactions all agents output the value 1 and continue

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doing so in all subsequent (infinite) computational steps. By replacing 1 with 0 we get the definition of the *reject* case. A GDM $\mathcal{A}$ *decides* a graph language $L \subseteq \mathcal{U}$ if it accepts any $G \in L$ and rejects any $G \notin L$, and a graph language is said to be *decidable* if some GDM decides it.

Theorem 1. *The class of decidable graph languages is closed under complement, union and intersection operations.*

Node and edge parity, bounded out-degree by a constant, existence of a node with more incoming than outgoing neighbors, and existence of some directed path of length at least $k = \mathcal{O}(1)$ are some examples of decidable graph languages, in the case where the graph universe is $\mathcal{G}$. Also, given that the graph universe is $\mathcal{G}$ one can prove the following.

Theorem 2. *There exists no GDM with stabilizing states to decide the graph language* $2C = \{G \in \mathcal{G} \mid G$ *has at least two nodes* u, υ *s.t. both* $(u,\upsilon),(\upsilon,u) \in E(G)$ *(in other words,* G *has at least one 2-cycle)*$\}$.

In the case where the graph universe is $\mathcal{H}$, containing all possible directed communication graphs (i.e. also the disconnected ones), we obtain the following strong impossibility results.

Lemma 1. *For any nontrivial graph language* L *(*L *is* nontrivial *if* $L \neq \emptyset$ *and* $L \neq \mathcal{H}$*), there exists some disconnected graph* G *in* L *where at least one component of* G *does not belong to* L *or there exists some disconnected graph* G' *in* $\overline{L}$ *where at least one component of* G' *does not belong to* $\overline{L}$ *(or both).*

Theorem 3. *Any nontrivial graph language* $L \subset \mathcal{H}$ *is undecidable by GDM.*

Corollary 1. *The graph language* $C = \{G \in \mathcal{H} \mid G$ *is (weakly) connected*$\}$ *is undecidable.*

Proof. C is a nontrivial graph language and Theorem 3 applies. □

A full version of this paper is available at http://fronts.cti.gr/aigaion/?TR=80

References

1. D. Angluin, J. Aspnes, Z. Diamadi, M.J. Fischer, and R. Peralta. Computation in networks of passively mobile finite-state sensors. In *23rd Annual ACM Sympsium on Principles of Distributed Computing* (*PODC*), pages 290-299, New York, NY, USA, 2004. ACM.
2. I. Chatzigiannakis, O. Michail, and P. G. Spirakis. Mediated Population Protocols. In *36th International Colloquium on Automata, Languages and Programming (ICALP)*, pages 363-374, Rhodes, Greece, 2009. (Also FRONTS Technical Report FRONTS-TR-2009-8, http://fronts.cti.gr/aigaion/?TR=65)
3. I. Chatzigiannakis, O. Michail, and P. G. Spirakis. Recent Advances in Population Protocols. In *34th International Symposium on Mathematical Foundations of Computer Science (MFCS)*, pages 56-76, Novy Smokovec, High Tatras, Slovak Republic, 2009. (Also FRONTS Technical Report FRONTS-TR-2009-21, http://fronts.cti.gr/aigaion/?TR=85)