

Principles of Computer Game Design and Implementation

Lecture 15

We already learned

- Collision Detection
 - two approaches (overlap test, intersection test)
 - Low-level, mid-level, and high-level view

Collision Response

- What happens after a collision is detected?
 1. Prologue
 - Check if collision should be ignored
 - Sound / visual effects
 2. Collision
 - *Resolve collision*
 3. Epilogue
 - Propagate the effects
 - destroy object(s), play sound...

Collision Resolution

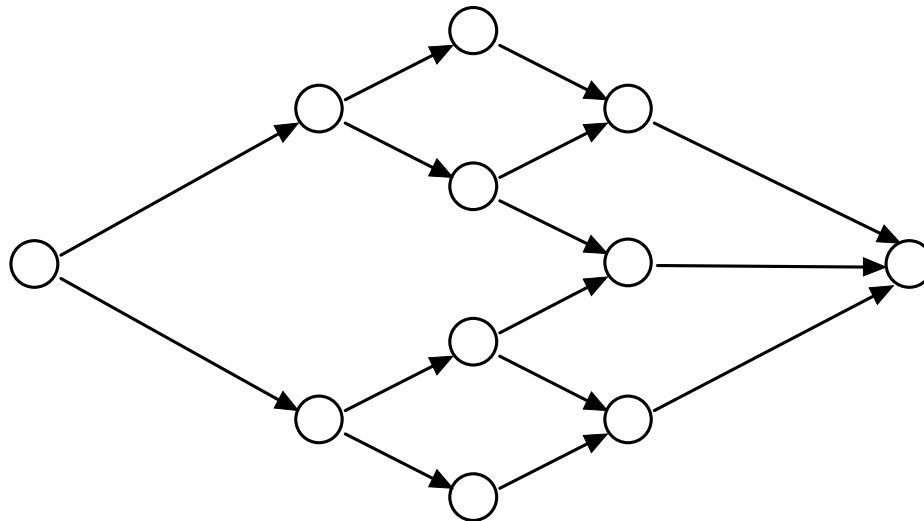
- Animation based
 - An artist models collision
 - A rocket hits a target...
 - Motion-capture
 - Sport games
- Physics based
 - Generated by an algorithm
 - Based on (more or less) realistic models



$a_{\dots} = \Delta v / \Delta t$	$F_{\dots} = ma$	$p = mv$
$v = v_0 + at$	$F_g = GMm/r^2$	$W = F \cdot \Delta s$
$\Delta s = v_0 t + \frac{1}{2} at^2$	$F_c = mv^2/r$	$P_{\dots} = \Delta W / \Delta t$
$V = IR$	$F_e = kq_1 q_2 / r^2$	$K = \frac{1}{2} mv^2$
$P = VI$	$F = qv \times B$	$U = mgh$
$R = \Sigma R$	$\tau = r \times F$	$\Delta U = Q - W$
$1/R_{\dots} = \Sigma 1/R$	$n = c/v$	$v = \lambda f$
$\epsilon_{\dots} = -N \frac{\Delta \Phi}{\Delta t}$	$n_1 \sin \theta_1 = n_2 \sin \theta_2$	$\frac{1}{f} = \frac{1}{d} + \frac{1}{d'}$
$\Delta x' = \Delta x / \gamma$	$E = mc^2$	$\Delta t' = \Delta t \gamma$

Recall: Classic Game Structure

- A convexity
- Starts with a single choice, widens to many choices, returns to a single choice

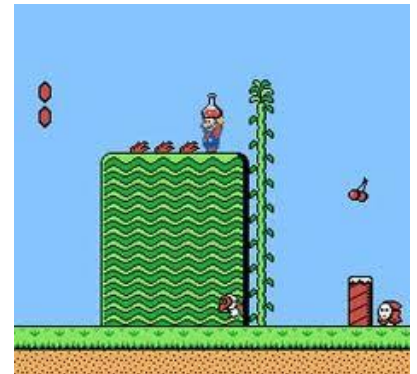


Why Physics?

- Responsive behaviour
 - Infinitely many possibilities
- For centuries people were *describing* the world
 - We can use the equations to *model* the world
- Can be hard
 - Knowledge of physics
 - “Real” physics is too expensive computationally

“Motion Science” in Games

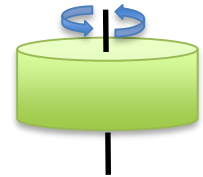
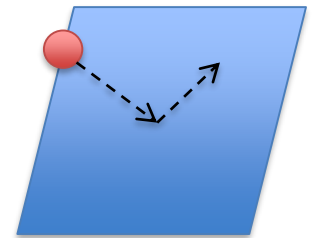
- Kinematics
 - Motion of bodies without considering forces, friction, acceleration,...
 - Not realistic
- Dynamics
 - Interaction with forces and torques



Keep It Simple

Separate translation and rotation

- Particle physics
 - A sphere with a perfect smooth, frictionless surface. No rotation
 - Interaction with forces and environment
 - Position, Velocity, Acceleration
- Solid body physics
 - Torques, angular velocity, angular momentum



Continuous Motion

- Particles move in a “smooth way”

- Position as a *function* of time

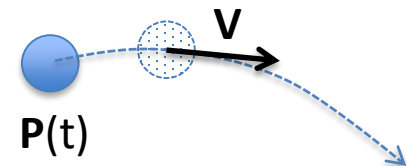
$\mathbf{P}(t)$ is the position of P in the moment t

- The *derivative*

$$\frac{d\mathbf{P}(t)}{dt}$$

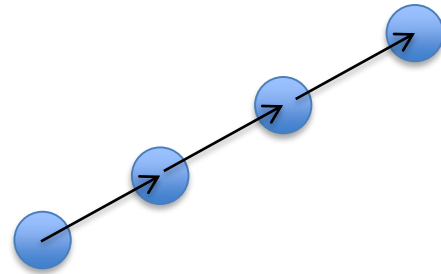
describes how $\mathbf{P}(t)$ changes over time

- Velocity (speed)

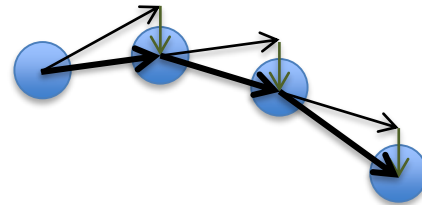


Discrete Particle Motion

- Uniform motion
 - Nothing affects the motion



- Gravitational pull



Integrators

- The process of computing the position of a body based on forces and interaction with other bodies is called *integration*
- A program that computes it is an *integrator*

Newton's Laws

1. Every body remains in a state of rest or uniform motion unless it is acted on by an external force

2. A body of mass m subject to force F accelerates as described by

$$F = ma$$

Vectors

3. Every action has an equal and opposite reaction

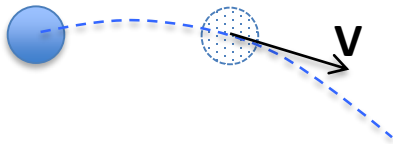
Position and Velocity

Continuous physics

- $\mathbf{V}(t) = \frac{d\mathbf{P}(t)}{dt}$



- $\mathbf{P}(t) = \dots$ (maths)



Discrete physics

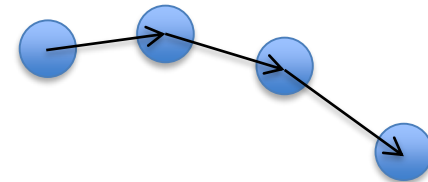
- $\mathbf{V}(t) = \frac{\Delta \mathbf{P}(t)}{\Delta t} = \frac{\mathbf{P}_{i+1} - \mathbf{P}_i}{tpf}$



- $\mathbf{P}_{i+1} = \mathbf{P}_i + tpf \cdot \mathbf{V}(t)$

Main loop iteration

Time per frame

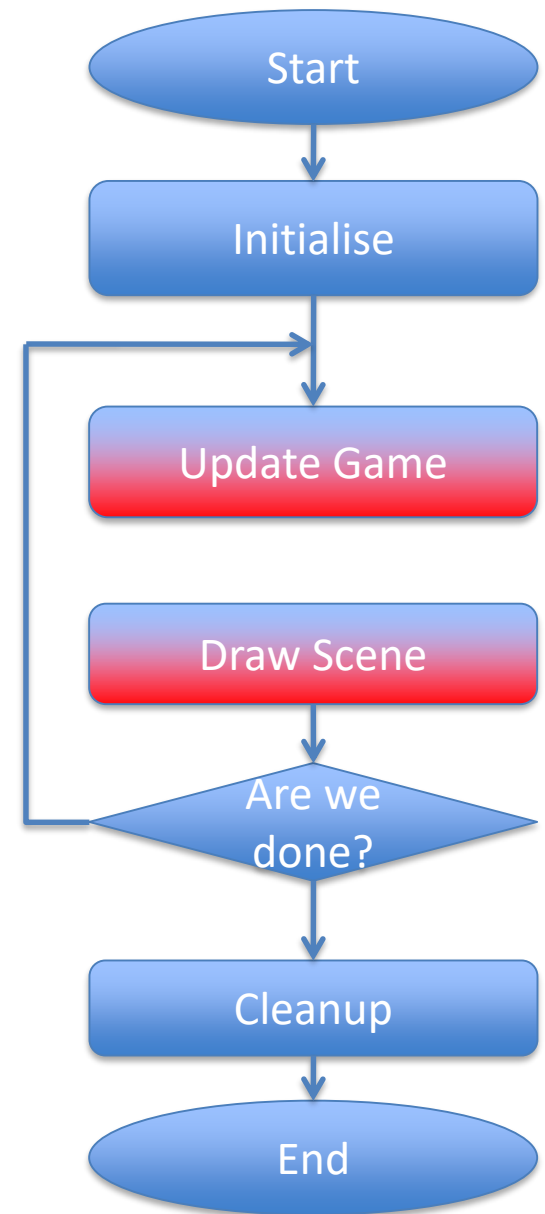


Recall: Arbitrary Translation

- Every iteration *update* the position

$$P = P + speed \cdot tpf \cdot \mathbf{U}(t)$$

- $\mathbf{U}(t)$ - the direction of movement
 - Depends on time!!
- *speed* is speed
- *tpf* is time per frame



Velocity and Acceleration

Continuous physics

- $\mathbf{a}(t) = \frac{d\mathbf{V}(t)}{dt}$



- $\mathbf{V}(t) = \dots(\text{maths})$

Main loop iteration

Discrete physics

- $\mathbf{a}(t) = \frac{\Delta\mathbf{V}(t)}{\Delta t} = \frac{\mathbf{V}_{i+1} - \mathbf{V}_i}{tpf}$

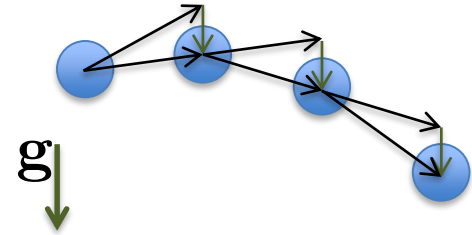


- $\mathbf{V}_{i+1} = \mathbf{V}_i + tpf \cdot \mathbf{a}(t)$

Time per
frame

Example: Gravitational Pull

- $\mathbf{a}(t) = \mathbf{g} = 9.8 \text{ N/kg}$
- $\mathbf{V}_{i+1} = \mathbf{V}_i + \text{tpf} \cdot \mathbf{g}$
- $\mathbf{P}_{i+1} = \mathbf{P}_i + \text{tpf} \cdot \mathbf{V}_{i+1}$



```
Vector3f velocity = new Vector3f(10,10,0);  
Vector3f gravity = new Vector3f(0, -9.8f, 0);  
...  
public void simpleUpdate() {  
    velocity = velocity.add(gravity(tpf));  
    ag.move(velocity.mult(tpf));  
}
```


Acceleration and Force

Newton's second law: a body of mass m subject to force \mathbf{F} accelerates as described by

$$\mathbf{F}(t) = m\mathbf{a}(t)$$



$$\mathbf{a}(t) = \mathbf{F}(t)/m$$

Example:

Engine thrust $\mathbf{F}_{\text{engine}} = k\mathbf{U}_V$

Linear drag $\mathbf{F}_D(t) = -b\mathbf{V}(t)$

Quadratic drag $\mathbf{F}_{\text{QD}}(t) = -c|\mathbf{V}(t)|^2\mathbf{V}(t)$

Use more often for simplicity

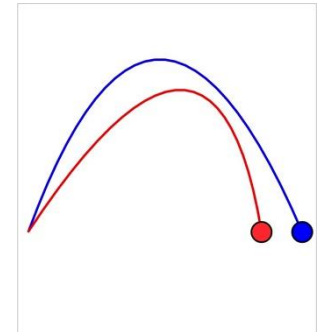
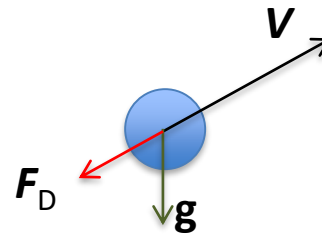
Example: Pull + Drag

$$\mathbf{F}_{i+1} = -b\mathbf{V}_i$$

$$\mathbf{a}_{i+1} = \mathbf{g} + \mathbf{F}_{i+1}/m$$

$$\mathbf{V}_{i+1} = \mathbf{V}_i + tpf \cdot \mathbf{a}_{i+1}$$

$$\mathbf{P}_{i+1} = \mathbf{P}_i + tpf \cdot \mathbf{V}_{i+1}$$



```
Vector3f force = velocityB.mult(-b);  
accelerationB = gravity.add(force.divide(m));  
velocityB =  
    velocityB.add(accelerationB.mult(tpf));  
bg.move(velocityB.mult(tpf));
```

Example: Pull + Drag + Thrust

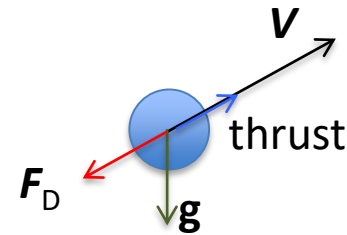
$$\mathbf{F}_{i+1} = -b\mathbf{V}_i + k\mathbf{U}_{\mathbf{V}}$$

Unit vector in the direction of \mathbf{V}

$$\mathbf{a}_{i+1} = \mathbf{g} + \mathbf{F}_{i+1}/m$$

$$\mathbf{V}_{i+1} = \mathbf{V}_i + tpf \cdot \mathbf{a}_{i+1}$$

$$\mathbf{P}_{i+1} = \mathbf{P}_i + tpf \cdot \mathbf{V}_{i+1}$$



```
Vector3f directionC = velocityC.normalize();
Vector3f forceC = velocityC.mult(-b).
                    add(directionC.mult(thrust));
accelerationC = gravity.add(forceC.divide(m));
velocityC = velocityC.add(accelerationC.mult(tpf));
cg.move(velocityC.mult(tpf));
```

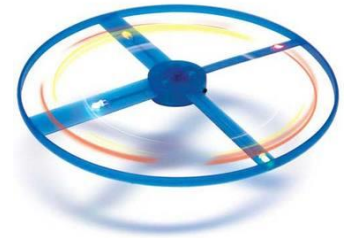
Simulation Recipe

- Add up all the forces acting on the object
 - Gravity, drag, thrust, spring pull,...
- Represent the motion as discrete steps

$$\left. \begin{aligned} \mathbf{a}_{i+1} &= \mathbf{g} + \mathbf{F}_{i+1}/m \\ \mathbf{V}_{i+1} &= \mathbf{V}_i + tpf \cdot \mathbf{a}_{i+1} \\ \mathbf{P}_{i+1} &= \mathbf{P}_i + tpf \cdot \mathbf{V}_{i+1} \end{aligned} \right\} \text{Euler steps}$$

Rotation

- Rotation of a uniform (again simplification) solid body can be described mathematically
 - Speed vs angular speed
 - Force vs torque
- Represent as discrete motion
- Use Euler steps to compute the rotation matrix
- Combine with translation



Hard but doable

Accuracy of Simulation

- How accurate this simulation is?
- Does it matter?
 - It's all about illusion, if the behaviour looks right, we do not care.
- But...