# Principles of Computer Game Design and Implementation 

Lecture 17

## We already learned

- Collision response
- Newtonian mechanics
- An application of Newtonian dynamics in targeting
- Collision recipe
- Ball-plain bouncing problem


## Demo



## Outline for today

- Collision recipe
- Ball-ball collision problem
- Other physics simulation
- rigid-body physics, soft-body physics, fluid mechanics, etc.
- A few examples for assignment 1


## Ball-Ball Collision Recipe

- First, consider 1D case

- No roll
- No friction
- No energy loss

Elastic collision

- Then 3D


## 1D Ball-Ball Collision Laws

- Impulse conservation
- Energy conservation

$$
\frac{m_{1} \mathrm{~V}_{1}^{2}}{2}+\frac{m_{2} \mathrm{~V}_{2}^{2}}{2}=\frac{m_{1} \mathrm{~V}_{1}^{\prime 2}}{2}+\frac{m_{2} \mathrm{~V}_{2}^{\prime 2}}{2}
$$



## 1D Ball-Ball Collision: Different Masses

- Can be solved

$$
\begin{aligned}
& V_{1}^{\prime}=\frac{V_{1}\left(m_{1}-m_{2}\right)+2 m_{2} V_{2}}{m_{1}+m_{2}} \\
& V_{2}^{\prime}=\frac{V_{2}\left(m_{2}-m_{1}\right)+2 m_{1} V_{1}}{m_{1}+m_{2}}
\end{aligned}
$$

## 1D Ball-Ball Collision: Same Mass

- If the balls have same mass (e.g. billiard balls)

$$
V_{1}^{\prime}=V_{2} \quad V_{2}^{\prime}=V_{1}
$$

Examples:

$$
\begin{aligned}
& V_{1}=10 \mathrm{mph}, V_{2}=0 \\
& V_{1}=10 \mathrm{mph}, V_{2}=-10 \mathrm{mph} \\
& V_{1}=10 \mathrm{mph}, V_{2}=3 \mathrm{mph}
\end{aligned}
$$

$$
\mathrm{V}_{1}^{\prime}=0, \mathrm{~V}_{2}^{\prime}=10 \mathrm{mph}
$$

$$
\mathrm{V}_{1}^{\prime}=-10 \mathrm{mph}, \mathrm{~V}_{2}^{\prime}=10 \mathrm{mph}
$$

$$
\mathrm{V}_{1}^{\prime}=3 \mathrm{mph}, \mathrm{~V}_{2}^{\prime}=10 \mathrm{mph}
$$

## Negative speed

 means that the ball moves from right to left

## Ball-Ball Inter Penetration



- $\mathrm{V}_{1}=10 \mathrm{mph}, \mathrm{V}_{2}=-10 \mathrm{mph}$
$\mathrm{V}_{1}{ }_{1}=-10 \mathrm{mph}, \mathrm{V}_{2}=10 \mathrm{mph}$
- $\mathrm{V}_{1}=-10 \mathrm{mph}, \mathrm{V}_{2}=10 \mathrm{mph} \quad \mathrm{V}_{1}=10 \mathrm{mph}, \mathrm{V}_{2}=-10 \mathrm{mph}$
- $\mathrm{V}_{1}=10 \mathrm{mph}, \mathrm{V}_{2}=-10 \mathrm{mph} \quad \mathrm{V}_{1}=-10 \mathrm{mph}, \mathrm{V}_{2}=10 \mathrm{mph}$
- $\mathrm{V}_{1}=-10 \mathrm{mph}, \mathrm{V}_{2}=10 \mathrm{mph} \quad \mathrm{V}_{1}=10 \mathrm{mph}, \mathrm{V}_{2}=-10 \mathrm{mph}$

Move nowhere!

## Ball-Ball Collision: Better Solution

- If $\left(\mathrm{V}_{1}-\mathrm{V}_{2}>0\right)$

$$
V_{1}^{\prime}=V_{2} \quad V_{2}^{\prime}=V_{1}
$$

- Else no change in velocities

- $\mathrm{V}_{1}=10 \mathrm{mph}, \mathrm{V}_{2}=-10 \mathrm{mph}$
$\mathrm{V}_{1}^{\prime}=-10 \mathrm{mph}, \mathrm{V}_{2}=10 \mathrm{mph}$
- $\mathrm{V}_{1}=-10 \mathrm{mph}, \mathrm{V}_{2}=10 \mathrm{mph}$
$\mathrm{V}^{\prime}{ }_{1}=10 \mathrm{mph}, \mathrm{V}^{\prime}=-10 \mathrm{mph}$
- $\mathrm{V}_{1}=10 \mathrm{mph}, \mathrm{V}_{2}=-10 \mathrm{mph}$
$\mathrm{V}^{\prime}{ }_{1}=-10 \mathrm{mph}, \mathrm{V}^{\prime}{ }_{2}=10 \mathrm{mph}$
- $\mathrm{V}_{1}=-10 \mathrm{mph}, \mathrm{V}_{2}=10 \mathrm{mph}$
$\mathrm{V}^{\prime}{ }_{1}=10 \mathrm{mph}, \mathrm{V}^{\prime}=-10 \mathrm{mph}$


## 3D Ball-Ball Collision (Same Mass)

- Collision does not change the parallel component of velocity

$$
\mathbf{N}=\frac{1}{\left\|\mathbf{P}_{2}-\mathbf{P}_{1}\right\|}\left(\mathbf{P}_{2}-\mathbf{P}_{1}\right)
$$

$$
\begin{array}{ll}
\mathbf{V}_{1 \mathbf{N}}=\left(\mathbf{N} \cdot \mathbf{V}_{1}\right) \mathbf{N} & \mathbf{V}_{2 \mathbf{N}}=\left(\mathbf{N} \cdot \mathbf{V}_{2}\right) \mathbf{N} \\
\mathbf{V}_{1| |}=\mathbf{V}_{1}-\mathbf{V}_{1 \mathbf{N}} & \mathbf{V}_{2| |}=\mathbf{V}_{1}-\mathbf{V}_{2 N}
\end{array}
$$



## Recall: Projection

- So,

$$
\operatorname{proj}_{\mathbf{V}} \mathbf{W}=\frac{\mathbf{W} \cdot \mathbf{V}}{\mathbf{V} \cdot \mathbf{V}} \mathbf{V}
$$



- If $\mathbf{V}$ is already normalized (often the case), then becomes

$$
\operatorname{proj}_{\mathbf{U}} \mathbf{W}=(\mathbf{W} \cdot \mathbf{U}) \mathbf{U}
$$

## 3D Ball-Ball Collision (Same Mass)

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$$
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$$

$$
\mathbf{V}_{1 \mathbf{N}}=\left(\mathbf{N} \cdot \mathbf{V}_{1}\right) \mathbf{N} \quad \mathbf{V}_{2 \mathbf{N}}=\left(\mathbf{N} \cdot \mathbf{V}_{2}\right) \mathbf{N}
$$

$$
\mathbf{V}_{1| |}=\mathbf{V}_{1}-\mathbf{V}_{1 \mathrm{~N}} \quad \mathbf{V}_{2 i!}=\mathbf{V}_{1}-\mathbf{V}_{2 \mathrm{~N}}
$$

$$
\mathbf{V}_{1 \mathbf{N}}^{\prime}=\left(\mathbf{N} \cdot \mathbf{V}_{2}\right) \mathbf{N} \quad \mathbf{V}_{2 \mathbf{N}}^{\prime}=\left(\mathbf{N} \cdot \mathbf{V}_{1}\right) \mathbf{N}
$$

$$
\mathbf{V}_{2}^{\prime}=\mathbf{V}_{1 N}^{\prime}+\mathbf{V}_{1 \| \mid} \quad \mathbf{V}_{2}^{\prime}=\mathbf{V}_{2 \mathrm{~N}}^{\prime}+\mathbf{V}_{2| |}
$$



## Same Mass Ball-Ball Collision jME code

```
if(...) {
```

    Vector3f \(n=\) ball2.getLocalTranslation().
                subtract(ball1.getLocalTranslation()).
                normalize();
    float proj1V = velocityl.dot(n);
    float proj2V = velocity2.dot(n);
    Vector3f \(\tan 1=\) velocity1.
                subtract(n.mult(proj1V));
    Vector3f tan2 = velocity2.
        subtract(n.mult(proj2V));
    if(proj1V - proj2V > 0) \{
        velocityl = tan1. add(n.mult(proj2V));
        velocity2 \(=\tan 2 . a d d(n . m u l t(p r o j 1 V)) ;\)
    
## Recall: Main Loop

Naïve approach:

$$
\begin{aligned}
& \text { for }(i=0 ; i<n u m \text { obj-1;i++) } \\
& \text { for }(j=i+1 ; j<\text { num_obj;j++) } \\
& \quad \text { if (collide(i,j))\{ } \\
& \quad \text { react; } \\
& \quad\}
\end{aligned}
$$



## Simple Newtonian Mechanics

- Accurate physical modelling can be quite complicated
- We considered simplest possible behaviours
- Particle motion
- Ball-plain and ball-ball collision
- No friction, no properties of materials


## Other Example: Box-Box collision

Boxes can interact in a number of ways


Hard to achieve a realistic behaviour without considering rotation, deformation, friction

## Other Physical Simulations



- Rigid body (no deformation) physics
- Rotation, friction, multiple collisions
- Joints and links
- Ragdoll physics



## More Physics

- Soft body physics (shapes can change)
- Cloth, ropes, hair

- Fluid dynamics



## Putting It All Together

- Combine all aspects of a physical model

I. Millington. Game Physics Engine Development.
- Use hardware acceleration


## Decoupling Physics and Graphics

- What if we need physics simulation for something not shown?
- E.g. reconsider the targeting problem

Drag acts on the projectile


## What Can We Do

- Euler steps give us the updated entity position based on the interaction with other entities and forces
- Analytical solution can be difficult to obtain
- Quadratic drag?
- Wind?
- Rocket-propelled grenade?


## Interactive Approach

- Compute the initial velocity as if there is no drag, wind, thrust,... (or simply pick a value)
- While not hit sufficiently close, repeat
- Use Euler steps to see where it gets
- If overshot, reduce speed
- If undershot, increase speed

Fun to watch, but does it solve our problem?

