# Principles of Computer Game Design and Implementation 

Lecture 6

## We already knew

- Game history
- game design information
- Game engine


## What's Next

- Mathematical concepts (lecture 6-10)
- Collision detection and resolution (lecture 1116)
- Game Al (lecture 17 - )


## Mathematical Concepts

3D modelling, model manipulation and rendering require Maths and Physics

- Typical tasks:
- How to position objects?
- How to move and rotate objects
- How do objects interact?


## 2D Space

- We will start with a 2D space (simpler) and look at issues involved in
- Modelling
- Rendering
- Transforming the model / view


## 2D Geometry

- Representation with two axes, usually X (horizontal) and $Y$ (vertical)
- Origin of the graph and of the 2D space is where the axes cross ( $\mathrm{X}=\mathrm{Y}=0$ )
- Points are identified by their coordinates



## Viewports

- A viewport (or window) is a rectangle of pixels representing a view into world space
- A viewport has its own coordinate system, which may not match that of the geometry.
- The axes will usually be $X$ horizontal \& $Y$ vertical
- But don't have to be - rotated viewports
- The scale of the axes may be different
- The direction of the $Y$ axis may differ.
- E.g. the geometry may be stored with Y up, but the viewport has $Y$ down.
- The origin (usually in the corners or centre of the viewport) may not match the geometry origin.


## Example

- Example of changing coordinate system from world space to viewport space:

$P=(20,15)$ in world space. Where is $P^{\prime}$ in viewport space?


## Rendering

- Rendering is the process of converting geometry into screen pixels
- To render a point:
- Convert vertex coordinates into viewport space
- Set the colour of the pixel at those coordinates
- The colour might be stored with the geometry, or we can use a fixed colour (e.g. black)


## Rendering Lines and Shapes

- Need to determine which part of the line is visible, where it meets the viewport edge and how to crop it.

- In "Ye good old days" this was rather difficult
- With support from rendering libraries easy


## Points and Vectors

- Point: a location in space
- Vector: a direction in space




## What's the Difference?

- The only difference is "meaning"
- But think about
- "move a picture to the right"
- "move a picture up"
- "move a picture in the direction ..."
- Vectors specify the direction


## Moving an Object

- Translation of an object
- Moving without rotating or reflecting
- Apply a vector to all points of an object
- Vector specifies direction and magnitude of translation





## Vectors

A vector is a directed line segment

- The length of the segment is called the length or magnitude of vector.
- The direction of the segment is called the direction of vector.
- Notations: vectors are usually denoted in bold type, e.g., $\mathbf{a}, \mathbf{u}, \mathbf{F}$, or underlined, $\underline{a}, \underline{\mathbf{u}}, \underline{\mathrm{~F}}$.

Same direction, red is twice as long

## Translation Recipe

- In order to translate (move) an object in the direction given by a vector $\mathbf{V}$, move all points.



$$
\begin{aligned}
& \mathbf{V}=\left(x_{v}, y_{v}\right) \\
& P=\left(x_{p}, y_{p}\right)
\end{aligned}
$$

$$
P^{\prime}=\left(x_{p}+x_{v}, y_{p}+y_{v}\right)
$$

## Multiplying a Vector by a Number

- Multiplying a vector by a positive scalar (positive number) does not change the direction but changes the magnitude
- Multiplying by a negative number reverses the direction and changes the magnitude



## In Coordinates

- $\mathbf{V}=(x, y)$ a vector, $\lambda$ a number

$$
\lambda \cdot V=(\lambda x, \lambda y)
$$

Example:

$$
\begin{aligned}
& 2 \cdot(2,5)=(4,10) \\
& 0.7 \cdot(2,5)=(1.4,3.5) \\
& -2 \cdot(2,5)=(-4,-10)
\end{aligned}
$$

## From A to B

- Which vector should be applied to move a point from $\left(x_{A}, y_{A}\right)$ to $\left(x_{B}, y_{B}\right)$ ?

$$
\left(x_{B}-x_{A}, y_{B}-y_{A}\right)
$$



## Sum of Two Vectors

- Two vectors $\mathbf{V}$ and $\mathbf{W}$ are added by placing the beginning of $\mathbf{W}$ at the end of $\mathbf{V}$.



## In Coordinates

Let

- $\mathbf{V}=\left(x_{v}, y_{v}\right)$
- $\mathbf{W}=\left(x_{w}, y_{w}\right)$

Then

$$
\mathbf{V}+\mathbf{W}=\left(\mathrm{x}_{\mathrm{v}}+\mathrm{x}_{\mathrm{w}}, \mathrm{y}_{\mathrm{v}}+\mathrm{y}_{\mathrm{w}}\right)
$$

## Vector Difference

- $\mathbf{V}-\mathbf{W}=\mathbf{V}+(-1) \cdot \mathbf{W}$



## In Coordinates

Let

- $\mathbf{V}=\left(x_{v}, y_{v}\right)$
- $\mathbf{W}=\left(x_{w}, y_{w}\right)$

Then

$$
\mathbf{V}-\mathbf{W}=\left(\mathrm{x}_{\mathrm{v}}-\mathrm{x}_{\mathrm{w}}, \mathrm{y}_{\mathrm{v}}-\mathrm{y}_{\mathrm{w}}\right)
$$

## Applications

- Apply vector $\mathbf{V}$ to an object then apply $\mathbf{W}$
- Apply V + W
- Representing motion as a combination of two
- If V takes you to $\mathrm{A}, \mathbf{W}$ takes you to B , what takes from $A$ to $B$ ?
- Apply W - V
- Shooting, targeting


## From 2D to 3D

- 3D geometry adds an extra axis over 2D geometry
- This "Z" axis represents "depth"
- Can choose the "direction" of Z



## "Handedness"

- Use thumb ( X ), index finger $(\mathrm{Y})$ \& middle finger $(Z)$ to represent the axes
- Use your left hand and the axes are lefthanded, otherwise they are right-handed



## Left- vs Right-Handed

- In mathematics, traditionally, right-handed axes are used
- In computing:
- DirectX and several graphics applications use lefthanded axes
- OpenGL use right-handed

Neither is better, just a choice

## Vectors in 3D

- Still a directed interval
- $x, y$ and $z$ coordinates define a vector

- $\mathbf{V}=\left(x_{v}, y_{v}, z_{v}\right)$ a vector, $\lambda$ a number
$\lambda \cdot \mathbf{V}=\left(\lambda x_{v}, \lambda y_{v}, \lambda z_{v}\right)$
- $\mathbf{V}=\left(\mathrm{x}_{\mathrm{v}}, \mathrm{y}_{\mathrm{w}}, \mathrm{z}_{\mathrm{v}}\right) ; \mathbf{W}=\left(\mathrm{x}_{\mathrm{w}}, \mathrm{y}_{\mathrm{w}}, \mathrm{z}_{\mathrm{w}}\right)$
$\mathbf{V}+\mathbf{W}=\left(\mathrm{x}_{\mathrm{v}}+\mathrm{x}_{\mathrm{w}}, \mathrm{y}_{\mathrm{v}}+\mathrm{y}_{\mathrm{w}}, \mathrm{z}_{\mathrm{v}}+\mathrm{z}_{\mathrm{w}}\right)$
- $\mathbf{V}=\left(\mathrm{x}_{\mathrm{v}}, \mathrm{y}_{\mathrm{w}}, \mathrm{z}_{\mathrm{v}}\right) ; \mathbf{W}=\left(\mathrm{x}_{\mathrm{w}}, \mathrm{y}_{\mathrm{w}}, \mathrm{z}_{\mathrm{w}}\right)$

$$
\begin{equation*}
\mathbf{V}-\mathbf{W}=\left(x_{v}-x_{w}, y_{v}-y_{w}, z_{v}-z_{w}\right) \tag{28}
\end{equation*}
$$

## Vectors in jMonkeyEngine

- jME defines two classes for vectors
- Vector3f
- Vector2f
- Constructors
- Vector2f(float x, float y)
- Vector3f(float $x$, float $y$, float $z)$
- Lots of useful methods (see javadoc)


## Translation (setting position) in JME

 protected void simplelnitApp() \{ Geometry box =...;Vector3f $\mathbf{v =}=$ new Vector3f(1,2,0); box.setLocalTranslation(v);
rootNode.attachChild(box), ${ }^{\text {Position of an object }}$

## Translation And the Scene Graph

- Let's model a table

Boxes


## Boxes for Tabletop and Legs

Box tableTop $=$ new $\operatorname{Box}(10,1,10)$;
Box leg1 = new $\operatorname{Box}(1,5,1)$;

Geometry gTableTop = new
Geometry("TableTop", tableTop);
gTableTop.setMaterial(mat);
Geometry gLeg1 = new Geometry("Leg1", leg1); gLeg1.setMaterial (mat);

## Beware of Floats

- If you think that the table top is too thick and change

Box tableTop $=$ new $\operatorname{Box}(10,1$, 10);
to Box tableTop $=$ new $\operatorname{Box}(10,0.3,10)$; you will see an error:
The constructor Box(int, double, int) is undefined

## Use the " f " word! :)

$$
\begin{aligned}
& \text { Box tableTop = new Box(10, } \\
& 0.3 \mathbf{f}, 10) ;
\end{aligned}
$$

float

Many jME methods take "single precision" float numbers as input

No need "double precision"

## Position the legs

leg1.setLocalTranslation (7, 0, 7); leg2.setLocalTranslation (-7, 0, 7); leg3.setLocalTranslation (7, 0,-7); leg4.setLocalTranslation(-7, 0,-7);

Attach all to rootNode

## Oops...



## A Better Scene Graph



## What are "table" and "legs"

- Internal nodes

Node table $=$ new
Node ("Table");
Node legs = new
Node("Legs");
roonode


## Putting it Together


table.attachChild(tableTop); table.attachChild(legs)
rootNode.attachChild(table);

## But Does It Change the Picture?

No


## Transforms Are in All Nodes!

$$
\text { legs.move }(0,-5 f, 0) ;
$$



## Summary: Manipulation of Vectors



Vector difference


## Summary: Vector Arithmetic

- $\mathbf{V}=\left(\mathrm{x}_{\mathrm{v}}, \mathrm{y}_{\mathrm{v}}, \mathrm{z}_{\mathrm{v}}\right)$ a vector, $\lambda$ a number

$$
\lambda \cdot V=\left(\lambda x_{v}, \lambda y_{v}, \lambda z_{v}\right)
$$

- $\mathbf{V}=\left(\mathrm{x}_{\mathrm{w}}, \mathrm{y}_{\mathrm{v}}, \mathrm{z}_{\mathrm{v}}\right) ; \mathbf{W}=\left(\mathrm{x}_{\mathrm{w}}, \mathrm{y}_{\mathrm{w}}, \mathrm{z}_{\mathrm{w}}\right)$

$$
\mathbf{V}+\mathbf{W}=\left(\mathrm{x}_{\mathrm{v}}+\mathrm{x}_{\mathrm{w}}, \mathrm{y}_{\mathrm{v}}+\mathrm{y}_{\mathrm{w}^{\prime}} \mathrm{z}_{\mathrm{v}}+\mathrm{z}_{\mathrm{w}}\right)
$$

$\mathbf{V}-\mathbf{W}=\left(\mathrm{x}_{\mathrm{v}}-\mathrm{x}_{\mathrm{w}}, \mathrm{y}_{\mathrm{v}}-\mathrm{y}_{\mathrm{w}}, \mathrm{z}_{\mathrm{v}}-\mathrm{z}_{\mathrm{w}}\right)$

What about a product of $\mathbf{V}$ and $\mathbf{W}$ ?
And why?

## Summary: Vector Algebra

- $a+b=b+a$
(commutative law)
- $(\mathbf{a}+\mathrm{b})+\mathbf{c}=\mathbf{a}+(\mathrm{b}+\mathbf{c})$
(associative law)
- $a+0=a$
- $a+(-a)=0$
- $\lambda(\mu a)=(\lambda \mu) a$
- $(\lambda+\mu) a=\lambda a+\mu a$
- $\lambda(\mathbf{a}+\mathbf{b})=\lambda \mathbf{a}+\lambda \mathbf{b}$
- $1 \mathbf{a}=\mathbf{a}$

