Deep Learning: Functional View and Features

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Up to now,

- Traditional Machine Learning Algorithms
- Deep learning
  - Introduction to Deep Learning (history of deep learning, including perceptron, multi-layer perceptron, why now?, etc)

Today’s Topics

- Functional View of DNNs
- Learning Representations & Features

Functional View of DNNs

What is a function?

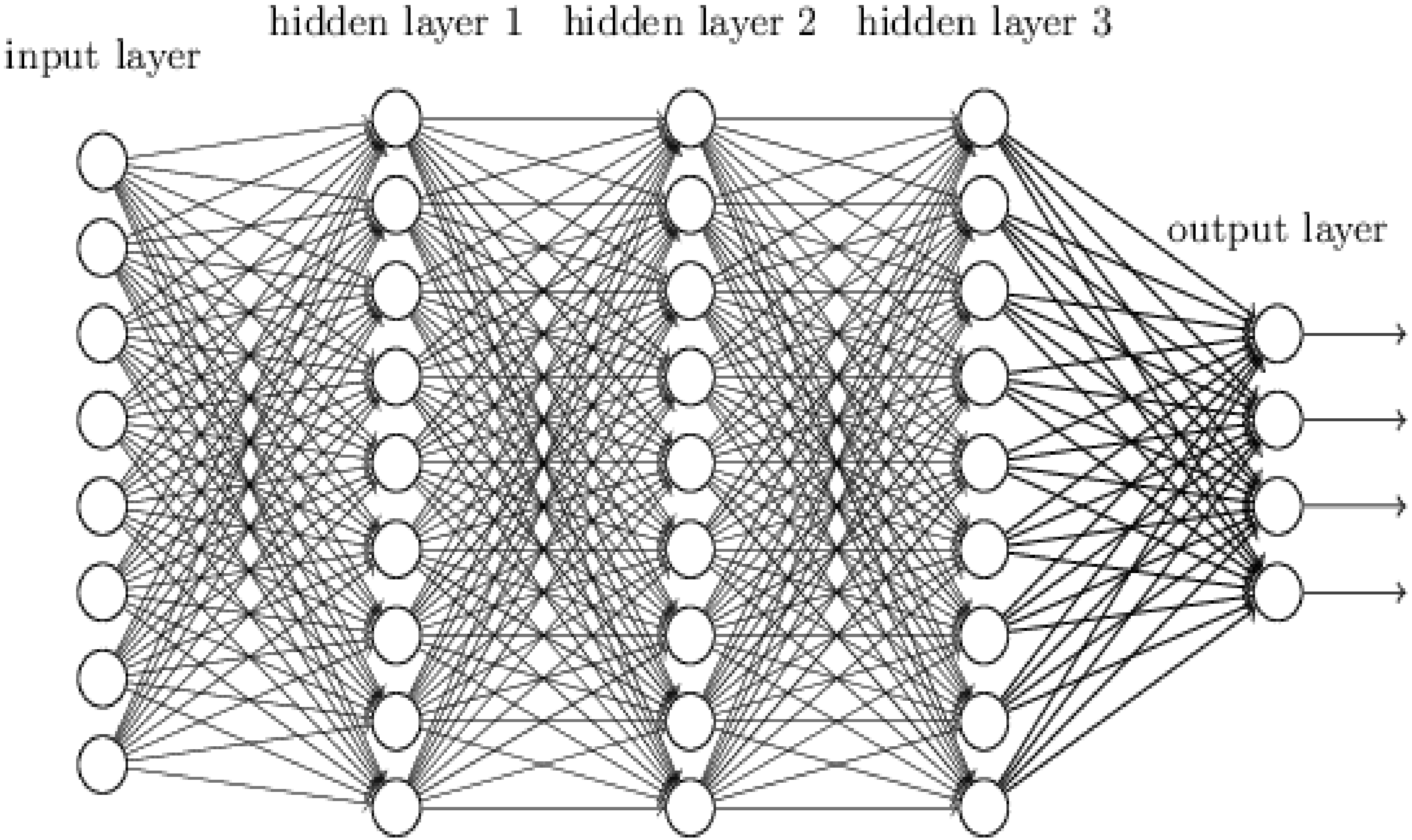
- In programming, a named section of a program that performs a specific task. In this sense, a function is a type of procedure or routine.
- The term *function* is also used synonymously with *operation* and command. For example, you execute the delete *function* to erase a word.

What is a **nested** function?

- a **nested function** (or **nested procedure** or **subroutine**) is a function which is defined within another function, the *enclosing function*.

```
float E(float x)
{
    float F(float y)
    {
        return x + y;
    }
    return F(3) + F(4);
}
```

A composition of a function $f \circ f$ with itself gives a nested function $f(f(x))$, $f \circ f \circ f$ which gives $f(f(f(x)))$, etc.


input layer
hidden layer 1
hidden layer 2
hidden layer 3
output layer

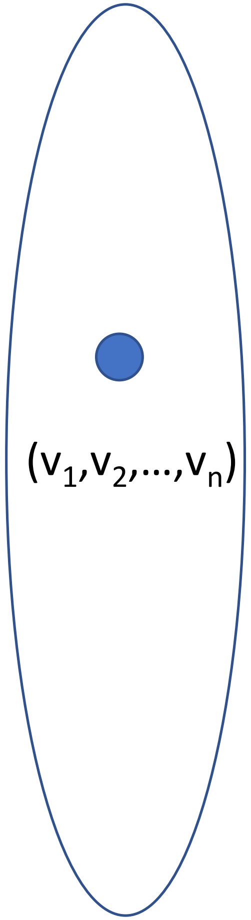
Functional View of DNNs

- A family of parametric, non-linear and hierarchical representation learning functions, which are massively optimized with stochastic gradient descent to encode domain knowledge, i.e. domain invariances, stationarity.

$$a_L(x;\theta_{1,\ldots,L}) = h_L\,(h_{L-1}(\ldots h_1(x,\theta_1),\theta_{L-1}),\ \theta_L)$$

x:input, θ_l: parameters for layer l, $a_l = h_l(x,\theta_l)$: (non-)linear function

Illustration of DNN function



Input space

Illustration of DNN function

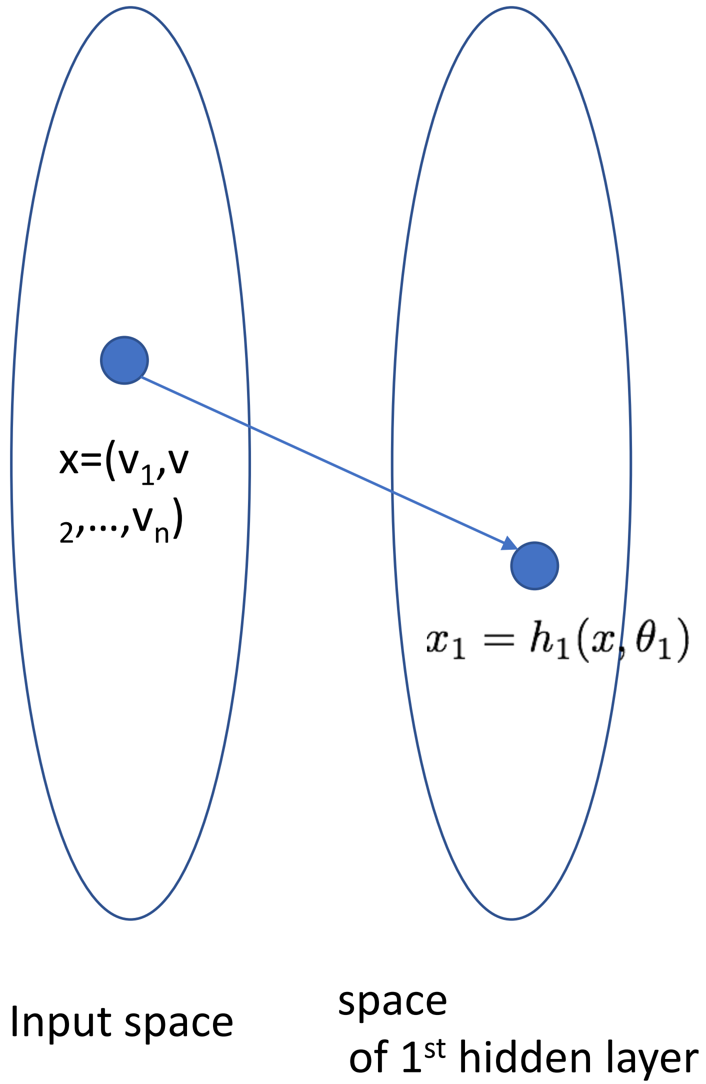


Illustration of DNN function

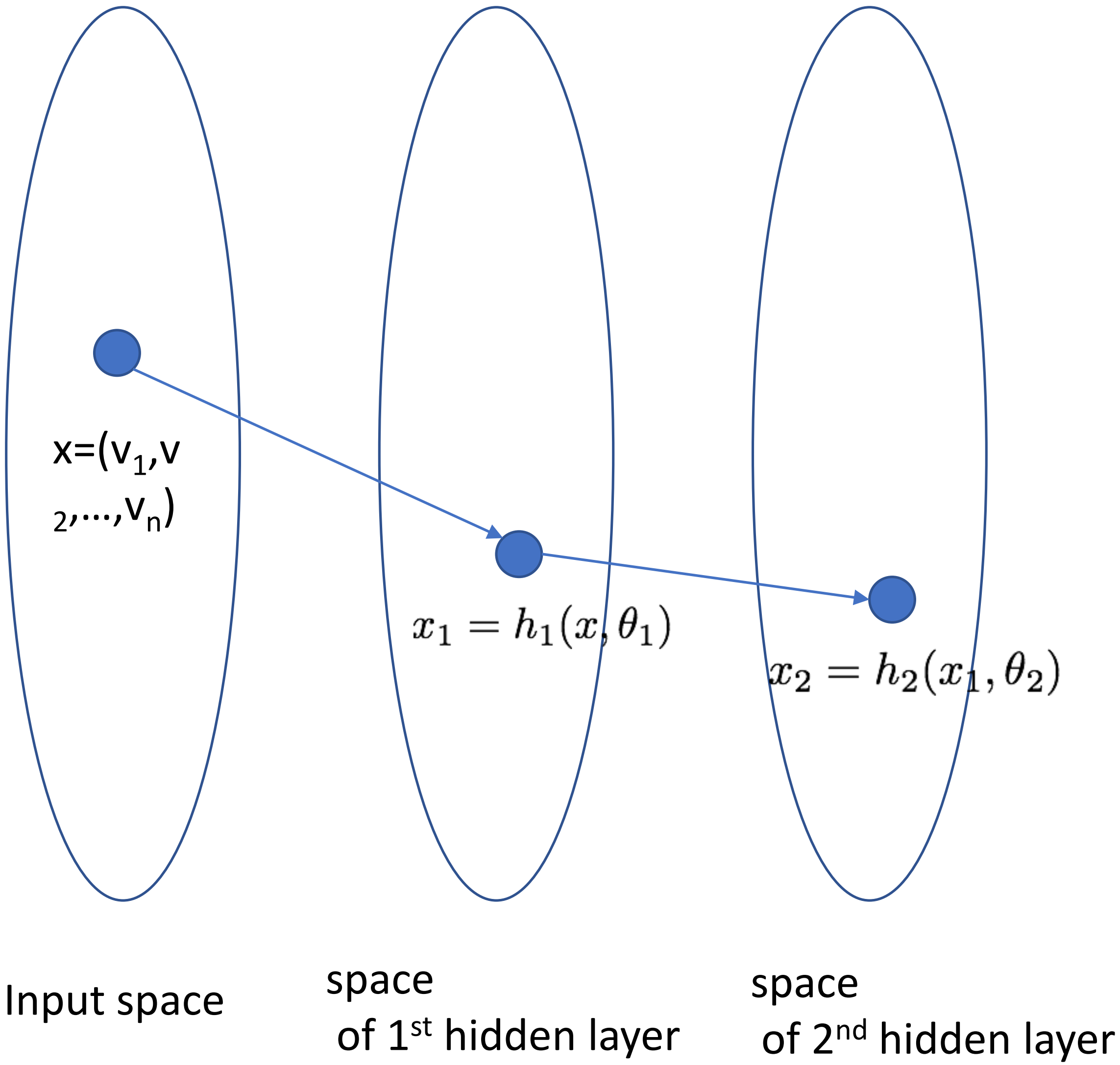
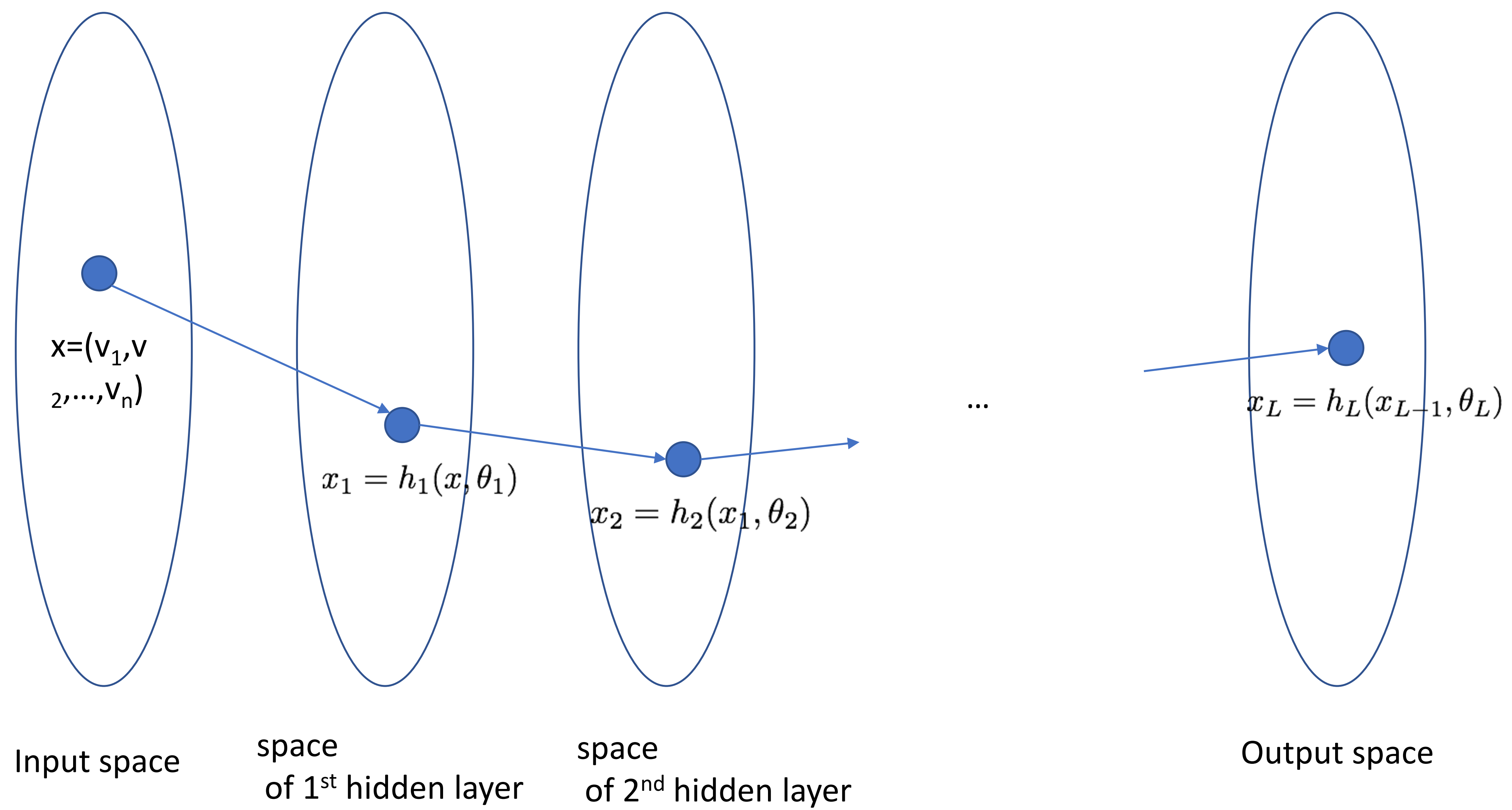


Illustration of DNN function



Note:

- Functions $h_1, h_2, \ldots, h_L$, are usually given.
- Parameters $\theta_1, \ldots, \theta_L$ are obtained by learning algorithm

Training Objective

Given training corpus $\{X, Y\}$ find optimal parameters

Find an optimal model parameterised over θ

Ground truth

Prediction

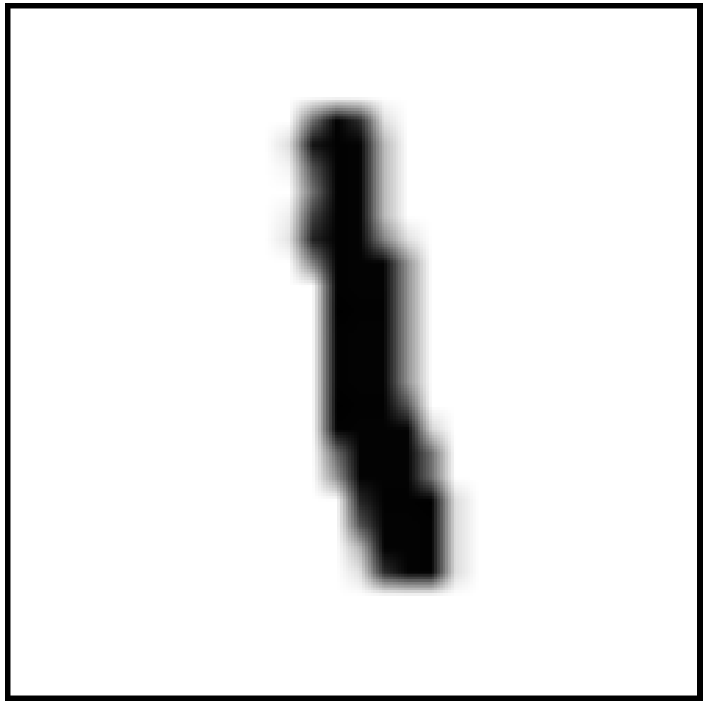
$$\theta^* \leftarrow \arg\min_\theta \sum_{(x,y)\subseteq(X,Y)} \ell(y, a_L(x; \theta_{1,\dots,L}))$$

accumulated loss

Loss function

Learning Representations & Features

Raw digital representation -- Image



$$\cong \begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&.6&.8&0&0&0&0&0&0\\
0&0&0&0&0&0&.7&1&0&0&0&0&0&0\\
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0&0&0&0&0&0&0&.9&1&.1&0&0&0&0\\
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0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix}$$

Raw digital representation -- Video

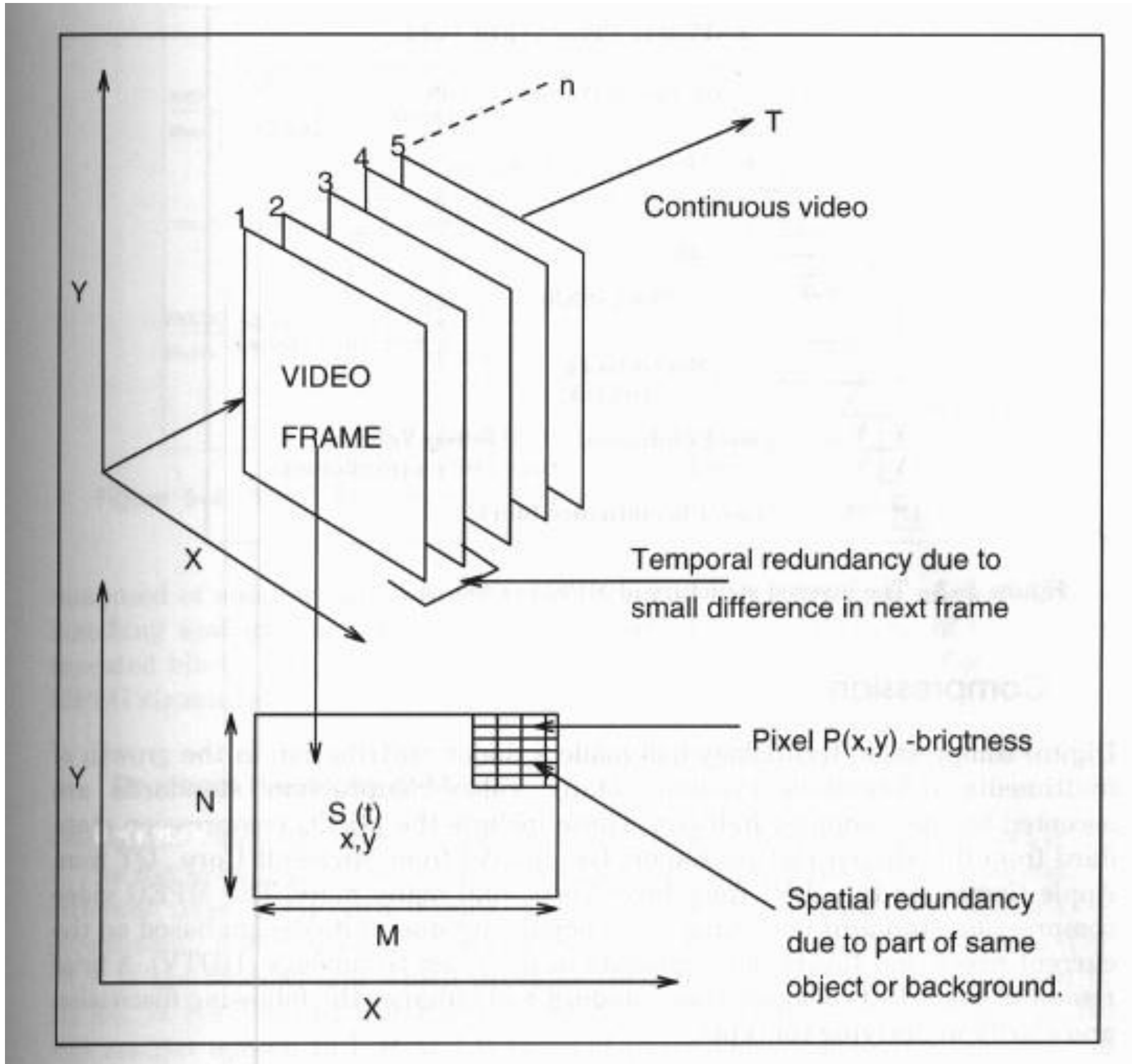
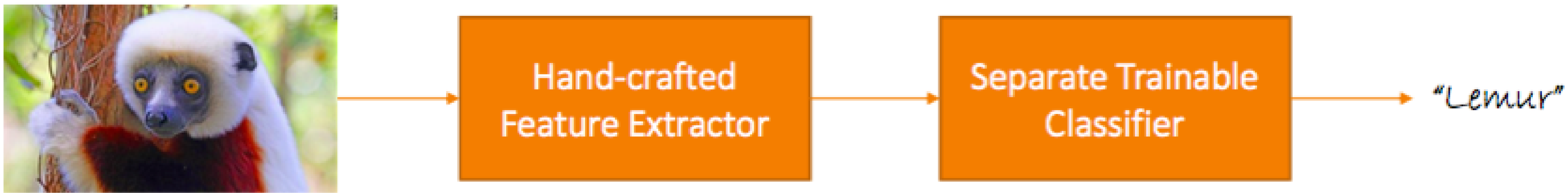


Figure 5-2 Representation of digital video.

Learning Representations & Features

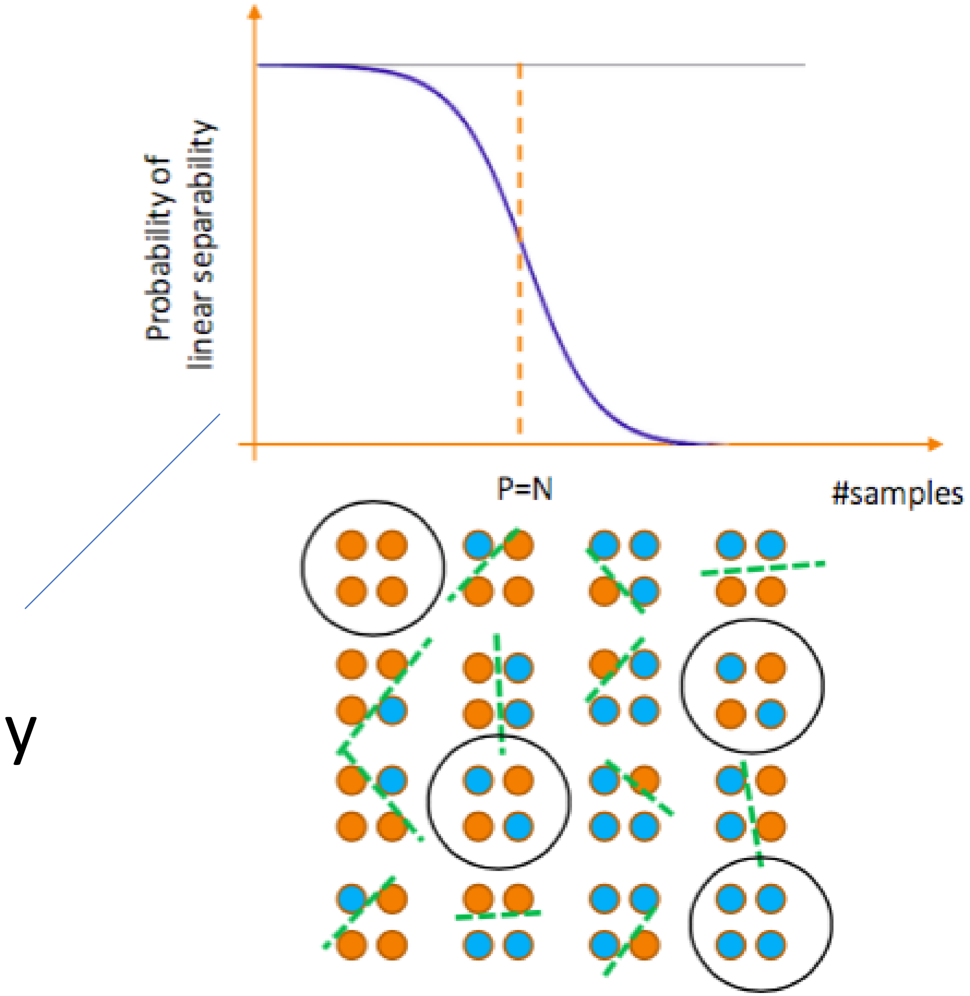
- Traditional pattern recognition



- End-to-end learning Features are also learned from data

Non-separability of linear machines

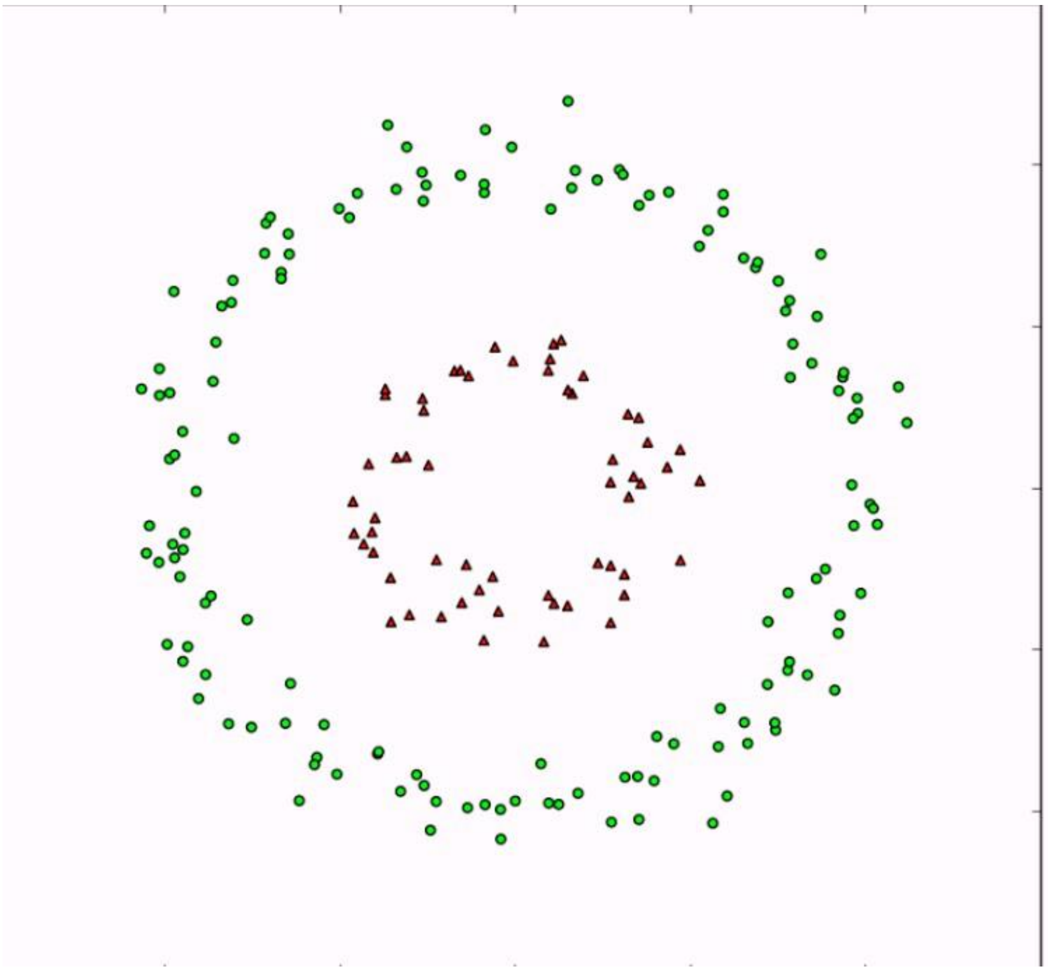
- $X = \{x_1, x_2, \ldots, x_n\} \in \mathrm{R}^d$
- Given the n points there are in total 2^n dichotomies
- Only about d are linearly separable
- With $n > d$ the probability X is linearly separable converges to 0 very fast
- The chances that a dichotomy is linearly separable is very small

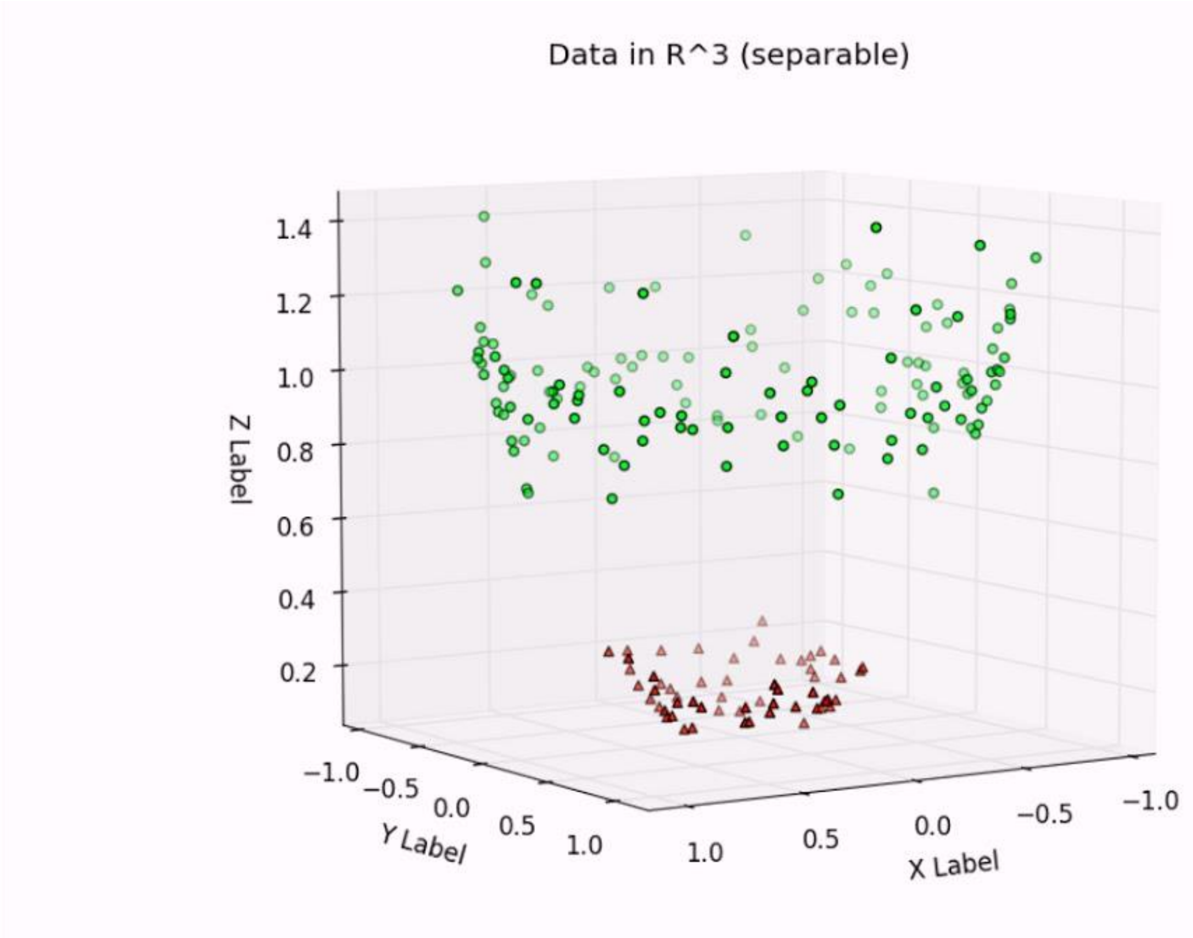


Non-linearizing linear machines

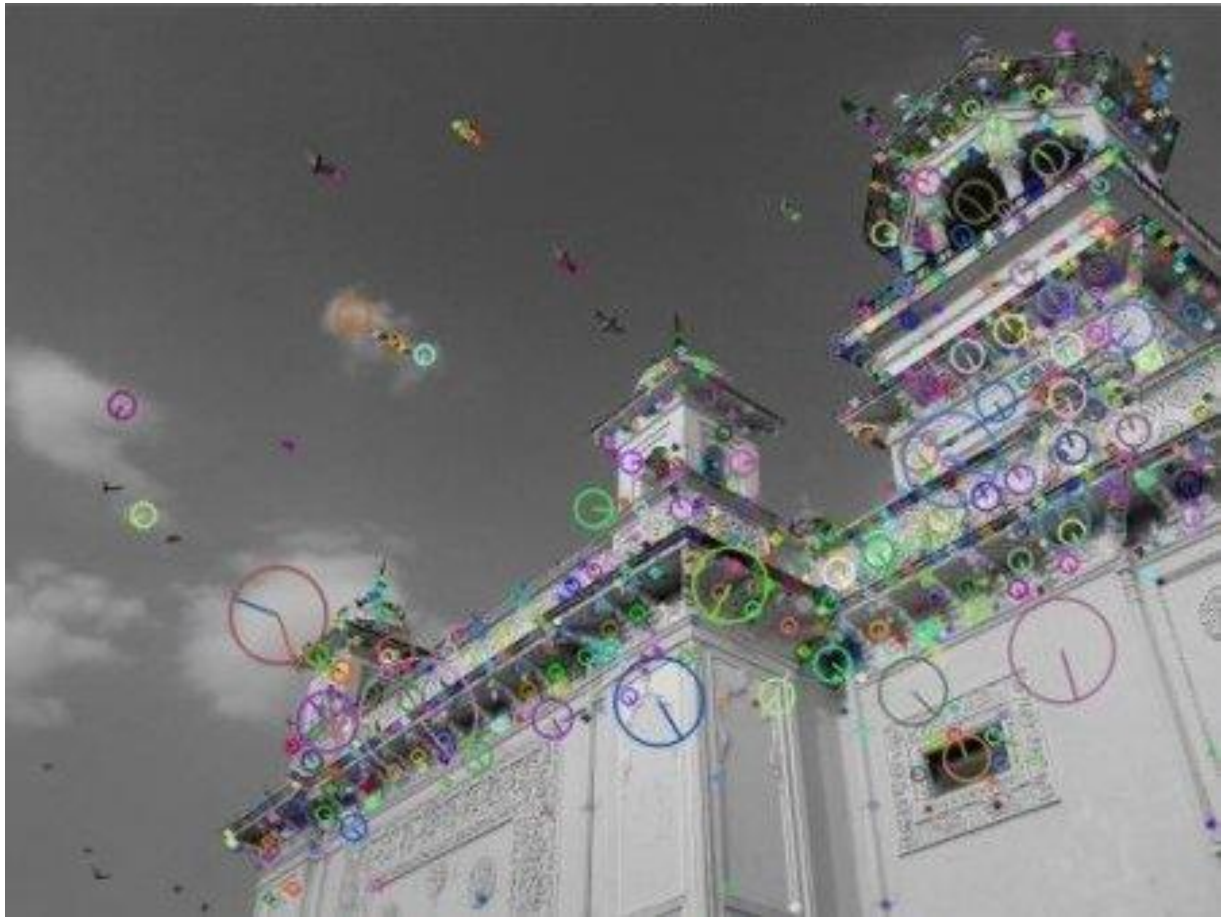
- Most data distributions and tasks are non-linear
- A linear assumption is often convenient, but not necessarily truthful
- Problem: How to get non-linear machines without too much effort?
- Solution: Make features non-linear
- What is a good non-linear feature?
  - Non-linear kernels, e.g., polynomial, RBF, etc
  - Explicit design of features (SIFT, HOG)?

Kernel: low dimension -> high dimension





SIFT

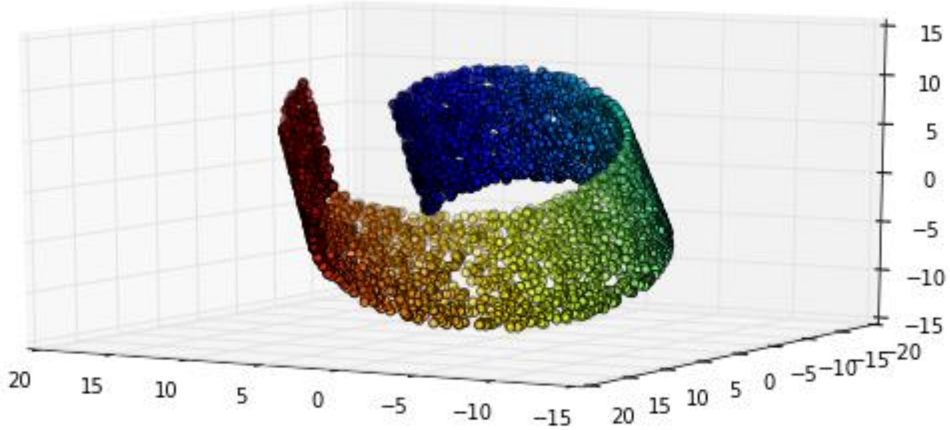


Good features

- Invariant
  - But not too invariant
- Repeatable
  - But not bursty
- Discriminative
  - But not too class-specific
- Robust
  - But sensitive enough

Data manifold

- High-dimensional data (e.g. faces) lie on lower dimensional manifolds



Every point represents an input sample.

- This is so-called "swiss roll". The data points are in 3d, but they all lie on 2d manifold, so the dimensionality of the manifold is 2, while the dimensionality of the input space is 3.

Data manifold is usually lower dimensional than the original

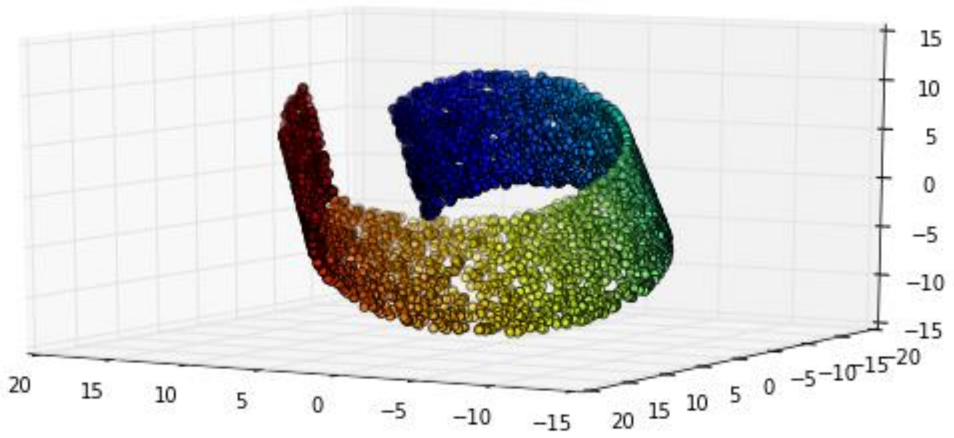
- High-dimensional data (e.g. faces) lie on lower dimensional manifolds
- Although the data points may consist of thousands of features, they may be described as a function of only a few underlying parameters.
  - That is, the data points are actually samples from a low-dimensional manifold that is embedded in a high-dimensional space.
- Goal: discover these lower dimensional manifolds
  - These manifolds are most probably highly non-linear

Hypothesis

- High-dimensional data (e.g. faces) lie on lower dimensional manifolds
  - Goal: discover these lower dimensional manifolds
  - These manifolds are most probably highly non-linear
- Hypothesis (1): Compute the coordinates of the input (e.g. a face image) to this non-linear manifold -> data become separable
- Hypothesis (2): Semantically similar things lie closer together than semantically dissimilar things

Hypothesis (1) -> existence of functional mapping

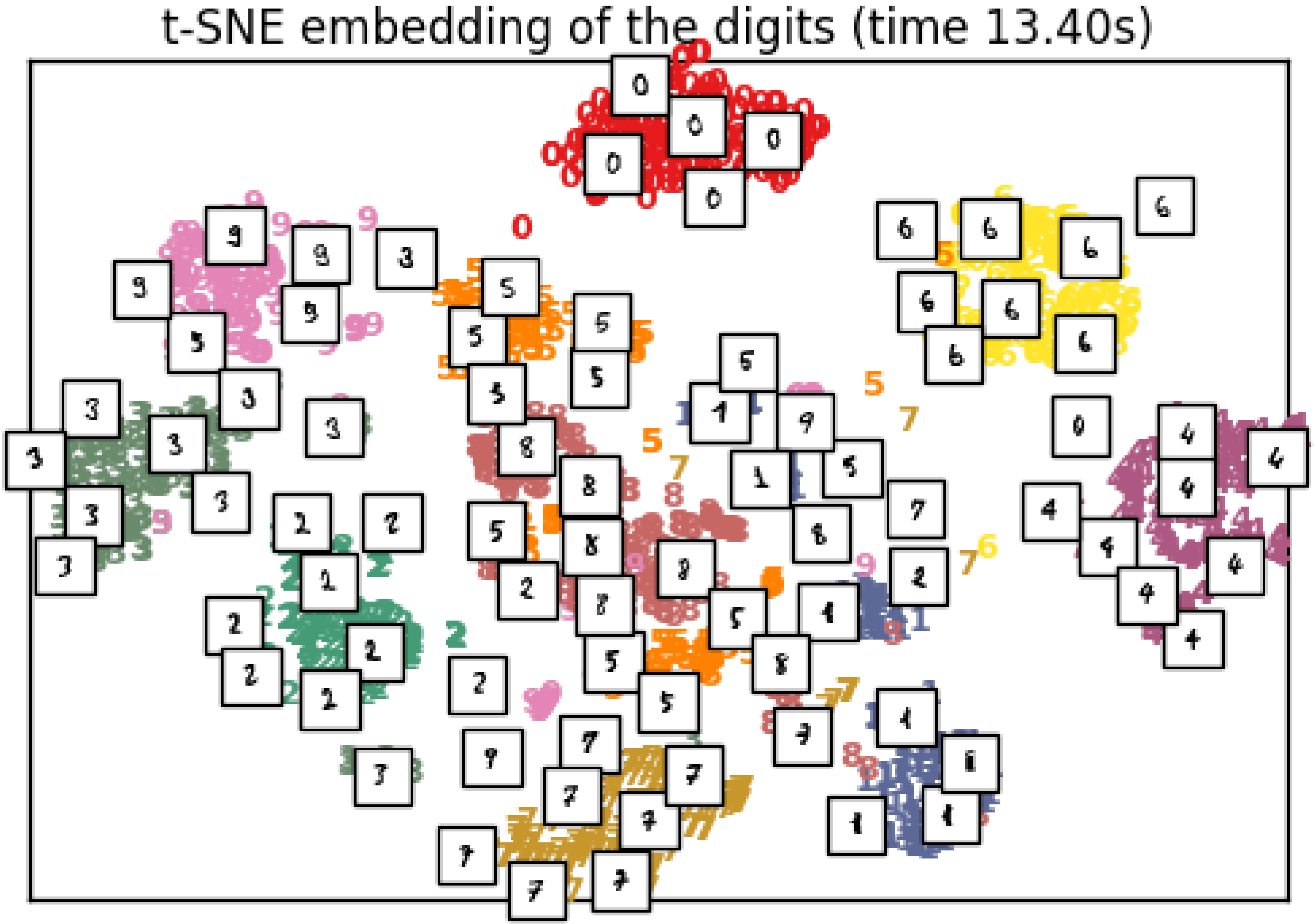
- High-dimensional data (e.g. faces) lie in lower dimensional manifolds



Every point represents an input sample.

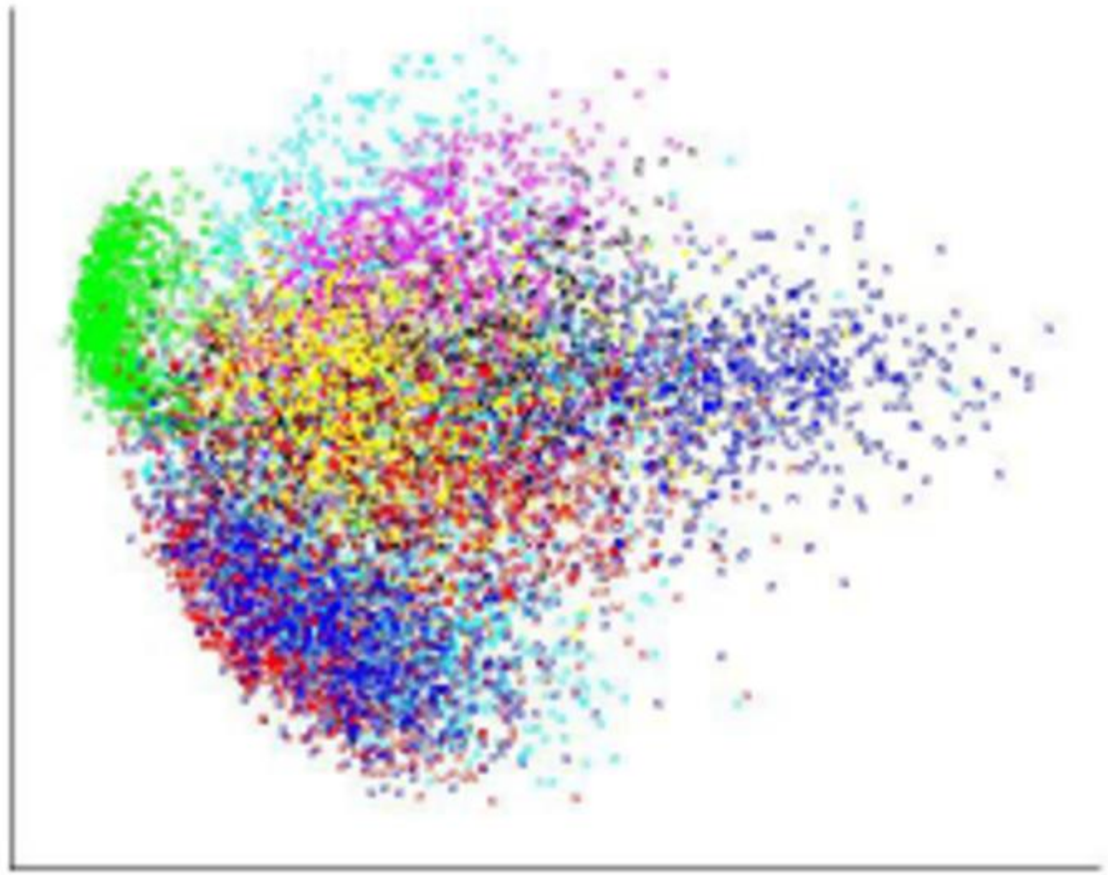
- So there should be a (non-linear) function mapping from 3d space to 2d space, on which the data can be linearly separable.

Hypothesis (2) -> some existing dimensional reduction methods



It is not linear, but can be largely separated with a dimensionality much less than 28*28

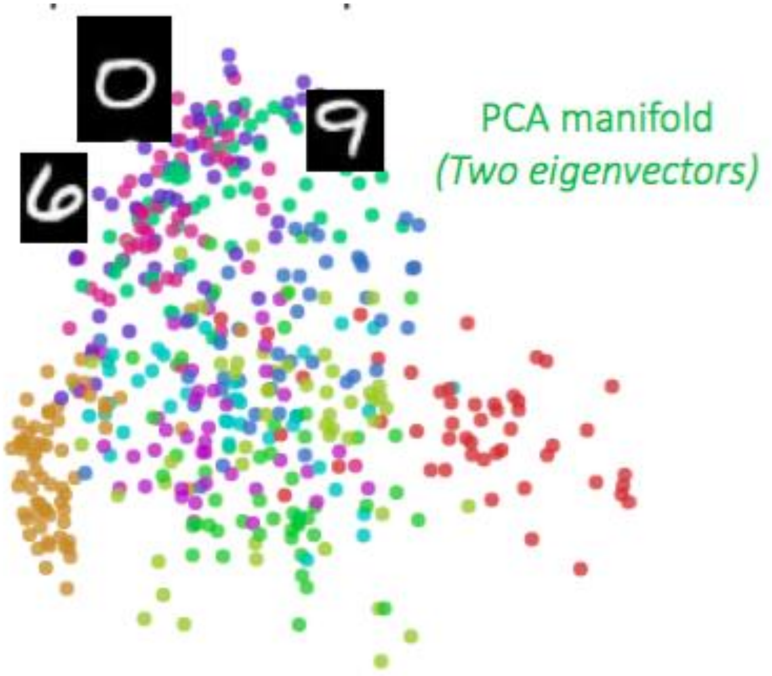
Hypothesis (2) -> some existing dimensional reduction methods

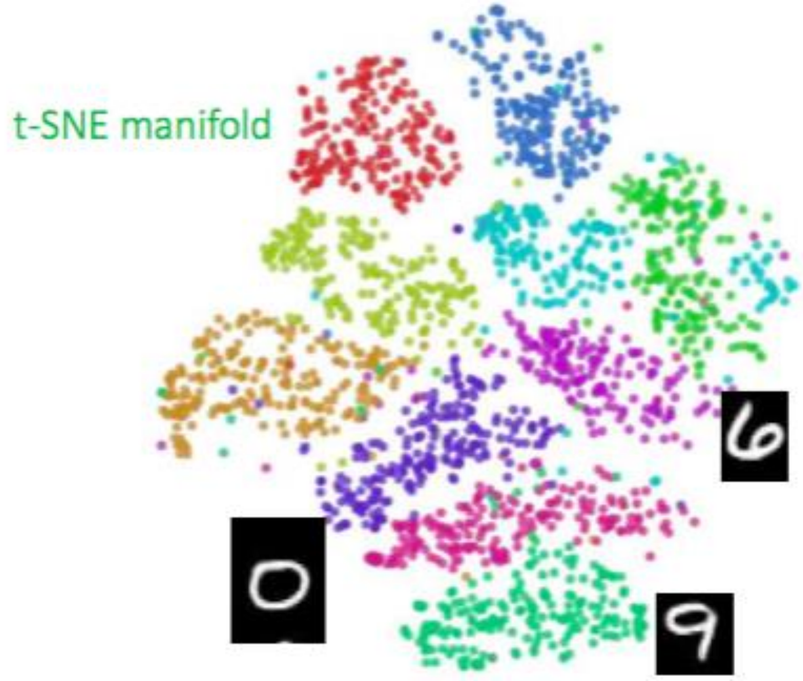


PCA (Principle Component Analysis), 2-dimensional

The digits manifolds

- There are good features and bad features, good manifold representations and bad manifold representations
- 28 pixels x 28 pixels = 784 dimensions



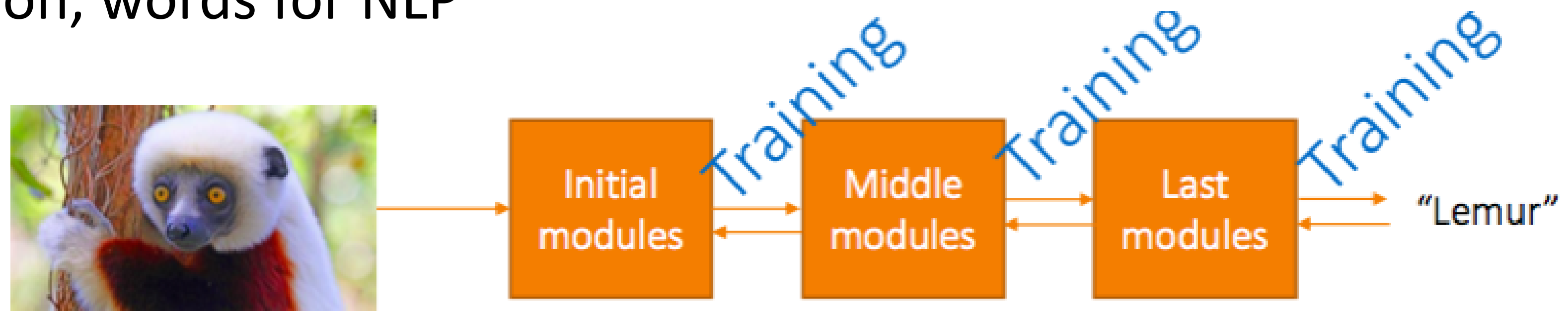


Difficulties of simply using dimensionality reduction or kernel

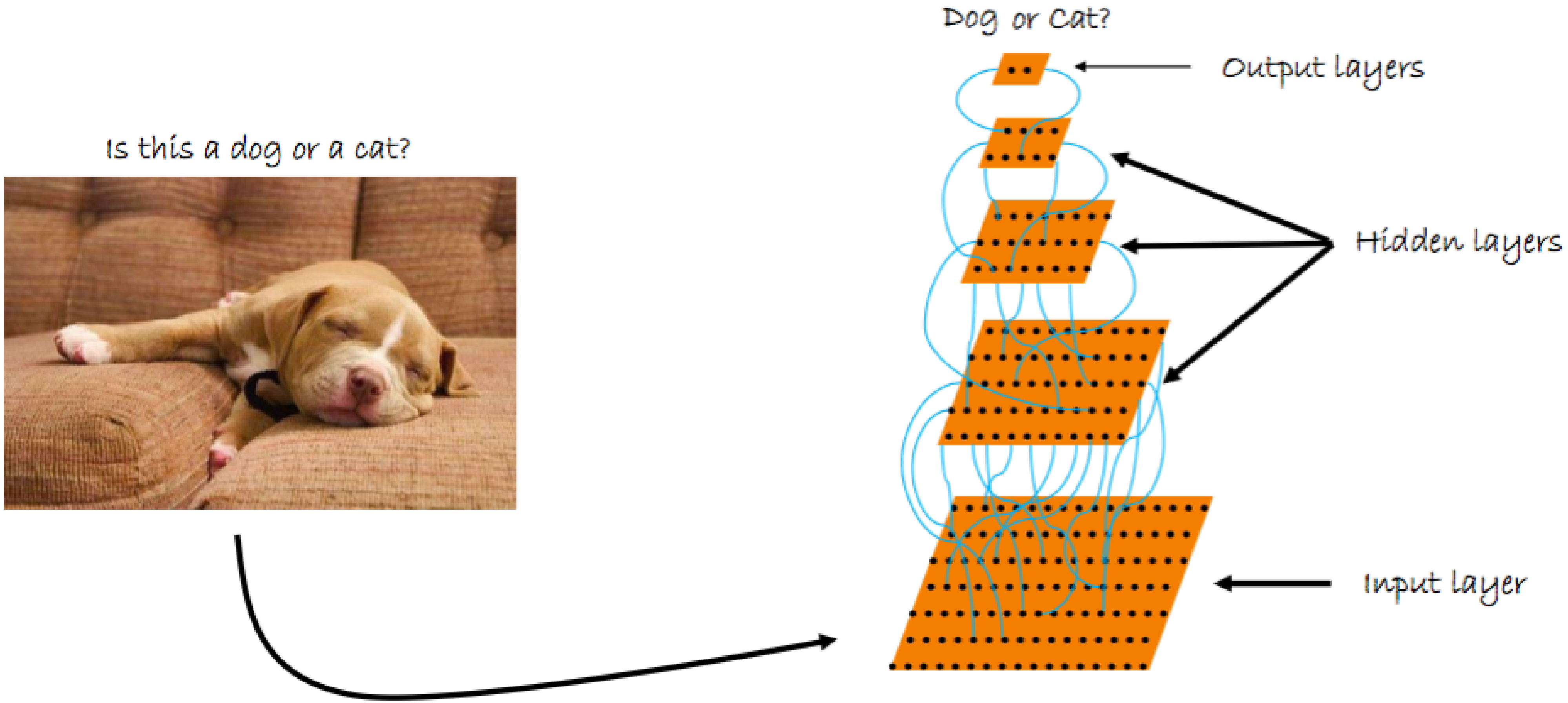
- Raw data live in huge dimensionalities
- Semantically meaningful raw data prefer lower dimensional manifolds
  - Which still live in the same huge dimensionalities
- Can we discover this manifold to embed our data on?

End-to-end learning of feature hierarchies

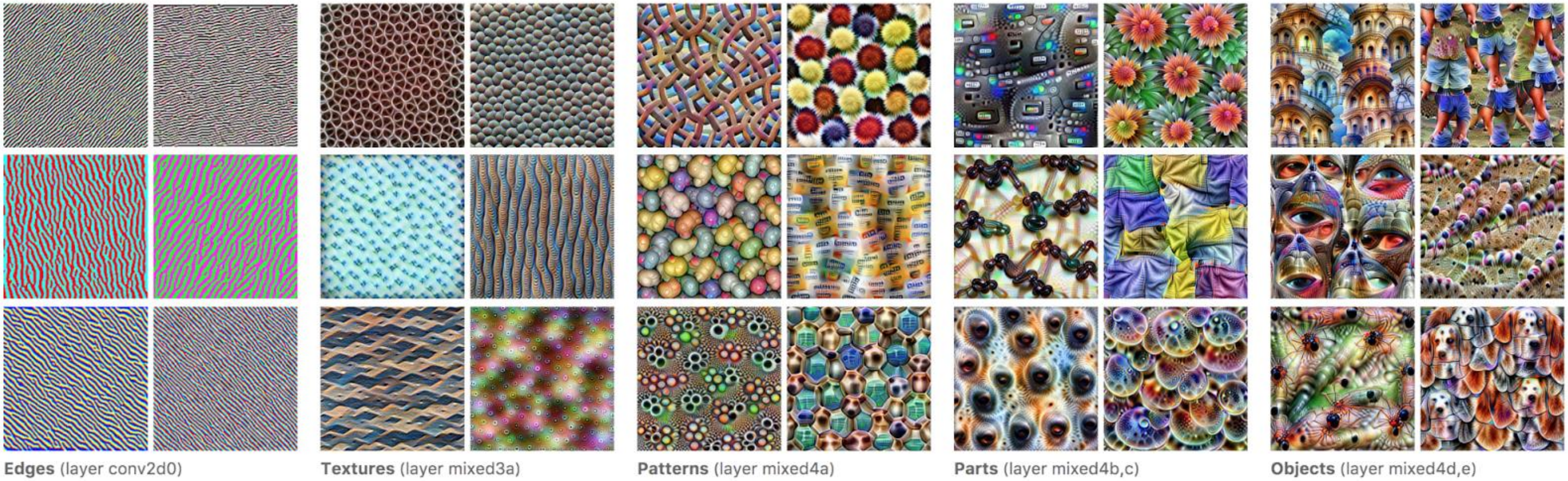
- A pipeline of successive modules
- Each module's output is the input for the next module
- Modules produce features of higher and higher abstractions
  - Initial modules capture low-level features (e.g. edges or corners)
  - Middle modules capture mid-level features (e.g. circles, squares, textures)
  - Last modules capture high level, class specific features (e.g. face detector)
- Preferably, input as raw as possible
  - Pixels for computer vision, words for NLP



Convolutional networks in a nutshell



Feature visualization by CNN



Why learn the features?

- Manually designed features
  - Often take a lot of time to come up with and implement
  - Often take a lot of time to validate
  - Often they are incomplete, as one cannot know if they are optimal for the task
- Learned features
  - Are easy to adapt
  - Very compact and specific to the task at hand
  - Given a basic architecture in mind, it is relatively easy and fast to optimize
- Time spent for designing features now spent for designing architectures